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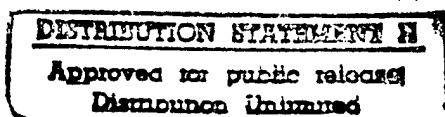
ARDENNES FRACTIONAL EXCHANGE RATIO RESEARCH

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13. ABSTRACT (Maximum 200 words) The US Army Concepts Analysis Agency (CAA) requires guidelines on use and computation of the combat measure of effectiveness (MOE) denoted as the fractional exchange ratio (FER). The Ardennes Campaign Simulation Data Base (ACSDB), derived from historical archives, records historical daily combat status for units engaged in the 1944-45 Ardennes Campaign of WWII. The objectives of this research effort are: (1) to survey users of simulation models at CAA to assess how FER is currently computed and used at CAA; (2) to use the ACSDB to develop insights on the empirical relationships of historical FER to other historical combat measures; and (3) to develop attributes of a preferred method for computing a basic FER at CAA. The ACSDB is used to derive empirical measures for FER, force ratio, and incremental force movement in the Ardennes Campaign. Statistical least-squares trendlines are then used to empirically relate FER to force ratio, and to force movement. These results, and the user survey, are used to develop guidelines on FER use at CAA.				
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January 1997

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**US Army Concepts Analysis Agency
8120 Woodmont Avenue
Bethesda, Maryland 20814-2797**

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ARDENNES FRACTIONAL EXCHANGE RATIO RESEARCH

SUMMARY
CAA-RP-97-1

THE REASON FOR PERFORMING THE RESEARCH is a need for empirical understanding of the meaning and utility of the combat measure of effectiveness (MOE) denoted as the fractional exchange ratio (FER). US Army Concepts Analysis Agency (CAA) also requires guidelines on basic FER definition and computation in order to establish a common frame of reference for simulation model users. The availability of the Ardennes Campaign Simulation Data Base (ACSDB) enables detailed empirical study of historical FERs associated with the WWII Ardennes Campaign of 1944-45.

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THE OBJECTIVES were to:

- (1) Determine FER formulation, computational methodology, and use in the Agency.
- (2) Use the ACSDB to develop insights on the empirical relationships of FER to other combat measures, and to develop the elements/attributes of a preferred method for computing FER.
- (3) Develop suggested guidelines for standardizing FER computation at CAA.

THE SCOPE OF THE RESEARCH was to use the ACSDB historical data to examine empirical relationships between the combat MOEs characterized by FER, force ratio, and progress of the forward edge of the battle area (FEBA). The derived relationships reflect, and apply to, the WWII Ardennes Campaign represented in the ACSDB. FER computational guidelines are limited in applicability to CAA combat simulations.

THE MAIN ASSUMPTIONS of this work are:

- (1) The ACSDB adequately represents the status and structure of forces in the actual WWII Ardennes Campaign of 1944-45.
- (2) Theater-size battles can be defined from the ACSDB with sufficient fidelity to derive meaningful associated FERs.

THE BASIC APPROACH was to:

- (1) Survey CAA model users to assess current practice in computation of FER.
- (2) Use the ACSDB to derive empirical FERs and force ratios and relate them to each other and to historical FEBA movement.

(3) Use empirical relationships and survey results to develop guidelines for computation of a standardized basic FER at CAA.

THE PRINCIPAL FINDINGS of the work reported herein are as follows:

(1) Since statistical investigations used sample historical data from only a single scenario, the derived observations and suggestions do not have a sufficient quantitative basis for generalization and should therefore be used as a basis for further testing rather than for implementation. The assessed ACSDB relationships between FER, force ratio, and FEBA movement include:

(a) **FER vs Force Ratio.** System FER and its associated force ratio, as reflected in the ACSDB, are strongly related by an exponential-form relationship when FER is based on losses in at least (combined) tanks, antitank (AT) weapons, and artillery systems. Personnel FER and force ratio also have a strong exponential-form relationship, if FER is based on only combat casualties (killed in action, captured/missing in action, and wounded in action). These exponential-form relationships may sufficiently characterize actual combat to be useful as a validation criterion for simulated combat in theater models.

(b) **FER vs FEBA Movement.** System FER is a rough indicator of approximate FEBA progress of engaged theater forces in the ACSDB. The strongest empirical relationships use FER computations with at least (combined) tanks, AT weapons, and artillery, with a damage criterion based on only destroyed and abandoned systems. Personnel FER based on combat casualties appears to be nearly as good an indicator of FEBA progress as system FER.

(c) **Force Ratio vs FEBA Movement.** Empirical results from the ACSDB suggest that force ratio is at least as good an indicator of approximate FEBA progress as is FER.

(2) The developed guidelines for computation of basic FER in CAA models include:

(a) A system FER based on losses in terms of destroyed and abandoned tanks, vehicular antitank weapons, attack helicopters, artillery, and mortars with diameter exceeding 120mm.

(b) A personnel FER based on losses in terms of combat casualties (killed in action, captured/missing in action, and wounded in action).

THE RESEARCH EFFORT was directed by Walter J. Bauman, Tactical Analysis Division, US Army Concepts Analysis Agency.

COMMENTS AND QUESTIONS may be sent to the Director, US Army Concepts Analysis Agency, ATTN: CSCA-TA, 8120 Woodmont Avenue, Bethesda, Maryland 20814-2797.

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CHAPTER 1

EXECUTIVE SUMMARY

1-1. BACKGROUND AND PROBLEM

a. Use of Fractional Exchange Ratio at CAA. The results of CAA combat simulations are presented in terms of measures of effectiveness (MOEs) which quantify the success of each combatant force at intervals during the simulated campaign. The fractional exchange ratio (FER) is a commonly used MOE which expresses the relative simulated performance/success of two opposing combatants in terms of each force's casualties or losses relative to their initial strength. While all users of FER agree on a single conceptual definition for that MOE, different users often apply that definition in different ways by selecting user-specific sets of target types and associated loss criteria for use in the FER calculation. The FERs computed by different users, while appearing descriptively to be the same, are difficult to compare when they are derived from different computational assumptions and attributes. Development of guidelines for a standardized computational definition for a preferred basic type of FER will establish a common frame of reference for CAA FER users. This paper addresses, among other things, a search for, and development of, standardization guidelines for a basic FER suitable for use at CAA.

b. Use of Historical Data to Improve Simulation Credibility. In 1995, CAA completed the Ardennes Campaign Simulation (ARCAS) Study (Ref. 1), a major simulation model validation effort. ARCAS compared a simulation of the 1944-45 Ardennes Campaign of World War II (WWII) with a large and detailed data base containing historical results of that campaign (Refs. 2, 3). Follow-on work proposed in the ARCAS effort included use of the data base of historical Ardennes battle results, denoted as the Ardennes Campaign Simulation Data Base (ACSDB), to derive and investigate statistical measures of combat which might be used to improve the credibility of combat simulation modeling at CAA. It has been conjectured that a close correlative relationship exists between FER and force ratio. The availability of the ACSDB enables this conjecture to be empirically investigated. It is also possible to investigate the relationship of historically-based FERs and force ratios to associated progress of the forward edge of the battle area (FEBA), as reflected in the ACSDB. There is currently no documentation on any history-based empirical FER relationship to force ratio or to combat movement. This paper derives historical FERs from the ACSDB and empirically analyzes their relationship to other combat measures, so that the usefulness of different types of FER computational methods can be assessed and used to select a preferred FER computation, which can enhance credibility of CAA simulation results.

1-2. PURPOSE AND OBJECTIVES

a. Purpose. The purpose of the Ardennes Fractional Exchange Ratio Research (ARFERR) effort is to investigate the use of FER at CAA, to empirically assess the relationship of FER to other combat measures, and to suggest guidelines for standardization of basic FER computation at CAA.

b. Objectives. The specific ARFERR objectives are to:

- (1) Determine FER formulation, computational methodology, and use in the Agency.
- (2) Use the ACSDB to develop insights on:
 - (a) The relationship of FER to force ratio.
 - (b) The relationship of FER to force combat effectiveness, as measured by FEBA movement.
 - (c) Elements/attributes of a preferred method for computing FER.
- (3) Develop suggested guidelines for standardizing FER use at CAA.

1-3. SCOPE

- a. Only FER use at CAA is addressed.
- b. All historical computations/statistics are derived from the WWII 1944-45 Ardennes Campaign, as represented in the ACSDB.
- c. Both personnel and weapon system losses are addressed in FER calculations from the ACSDB.
- d. Both permanent (destroyed and abandoned) and temporary (destroyed, abandoned, and damaged) losses are addressed in FER calculations.

1-4. LIMITATIONS

- a. FERs are computed only for the full Ardennes theater battle, and for the battle comprising the "bulge" within the ARCAS theater, as defined in the ARCAS Study.
- b. The only measures of historical force combat effectiveness examined are average FEBA movement in the Ardennes theater, and in the Ardennes bulge, as previously derived from the ACSDB in the ARCAS Study.
- c. Sample limitations on combat statistics derived from the ACSDB limit the generalization of suggested relationships developed in this effort. Consequently, any developed suggestions should be regarded as more suitable for further investigation than for implementation.
- d. Helicopters and tactical aircraft are not included in any FER computations derived from the ACSDB.
- e. Statistical processing of data is restricted to that which can be done using Microsoft EXCEL 5 software on a PC employing a Pentium processor.

1-5. ASSUMPTIONS

a. The ACSDB adequately represents the status and structure of forces in the actual WWII Ardennes Campaign of 1944-45.

b. Theater-size battles can be defined from the ACSDB with sufficient fidelity to derive meaningful associated FERs.

c. The spectrum of types of FER calculations chosen for analysis is sufficiently broad for useful differences to be assessed.

1-6. STUDY APPROACH AND METHODOLOGY. The study methodology sequentially executes the following phases.

a. **User Survey.** A model user survey, consisting of interviews with CAA simulation users, is used to assess and compare rules typically applied when computing FER from simulation results. Based on the results of these interviews, a cross-reference table is constructed showing similarities and differences among users in the elements and attributes used in their basic FER computations. The subject simulation models are EAGLE, the Combat Sample Generator (COSAGE), and the Concepts Evaluation Model IX (CEM IX).

b. **Analysis of Combat Statistics Derived from the ACSDB.** Data in the ACSDB are used to compute historical FER values, and force ratio values, for the Ardennes theater battle, and for the Ardennes Bulge battle, in eight 4-day time periods, based on four different weapon system mixes and two different loss criteria. Correlative empirical relationships are then sought between FER and force ratio, between FER and FEBA movement, and between force ratio and FEBA movement.

c. **Development of Recommendations for Computation of a Basic FER.** The responses of the model user survey, and results from the analysis of correlative relationships between FER, force ratio, and FEBA movement, as computed from the ACSDB, are used to develop guidelines and recommendations for a preferred computational procedure for a basic FER at CAA.

1-7. SUMMARY OF FINDINGS AND OBSERVATIONS

a. **Relationships Between FER and ACSDB Combat Statistics.** Table 1-1 summarizes the observations on assessed ACSDB relationships between FER, force ratio, and FEBA movement.

Table 1-1. ACSDB Relationships of FER to Force Ratio and FEBA Movement

Relationship	Usefulness of relationship	Preferred FER damage criteria
FER vs force ratio	Exponential relation may be useful for model validation.	Dmgd/dst/abnd systems, or combat casualties (personnel)
FER vs combat effectiveness (FEBA advance)	FER as advantage factor may indicate approximate FEBA move potential.	Dst/abnd systems, combat casualties (personnel)
Force ratio vs combat effectiveness (FEBA advance)	Force ratio is at least as useful as FER for indication of FEBA move potential.	Not applicable

(1) FER vs Force Ratio. Correlative statistical results suggest that system FER and its associated force ratio, as derived from, and reflected in, the ACSDB, are strongly related by an exponential-form relationship when FER is based on losses in at least (combined) tanks, antitank (AT) weapons, and artillery systems. Personnel FER and force ratio, as derived from the ACSDB, also have a strong exponential-form relationship, if FER is based on only combat losses (killed in action (KIA), wounded in action (WIA), captured/missing in action (CMIA)). The exponential-form relationship between FER and force ratio, when FER is based on at least damaged, destroyed, and abandoned (combined) tanks, AT weapons, and artillery, may sufficiently characterize actual combat to be useful as a validation criterion for simulated combat in theater models.

(2) FER vs Combat Effectiveness (FEBA movement) in ACSDB Cases. Correlative statistical results suggest that system FER is a rough indicator of approximate FEBA progress of engaged theater forces represented in the ACSDB. The strongest empirical relationships, based on both coefficient of variation and average fitting error, are associated with the ACSDB cases computing FER using at least (combined) tanks, AT weapons, and artillery with a damage criterion based on only destroyed and abandoned systems. Personnel FER based on combat casualties is nearly as good an indicator of FEBA progress as system FER and appears to be a slightly better indicator than personnel FER based on total casualties.

(3) Force Ratio vs Combat Effectiveness (FEBA movement) in ACSDB Cases. Empirical results from the ACSDB suggest that force ratio is at least as good an indicator of approximate FEBA progress as is FER. All system force ratios analyzed in the ACSDB data show very similar strength in their relationship to FEBA movement, based on both coefficient of variation and average fitting error.

b. Recommendations for Computation of a Basic FER. Table 1-2 summarizes suggested guidelines for standardized computation of a basic system FER and a basic personnel FER within CAA based on analytic results and correlative relationships, derived from the ACSDB, evaluated in light of the following assessment criteria:

- (1) Feasibility of generation/computation of the measure in CAA models.
- (2) Consistency in meaning of FER components, when used in different CAA models.
- (3) Reasonably strong relationship between FER and combat effectiveness (expressed as FEBA movement).

Table 1-2. Recommendations for Computation/Definition of Basic FER

Type FER	Systems included	Systems excluded	Losses included
System FER	Tanks, vehicular AT weapons, attack helicopters, mortars >120mm, artillery	Man-carried wpns, mortars < 121mm, tacair, AD wpns	Destroyed and abandoned weapons
Personnel FER	N/A	N/A	KIA, CMIA, WIA

If only one type FER is computed, the personnel FER appears to be preferred because it is more readily computed, uses homogeneous items, and, in the form of the advantage factor, has previously demonstrated an empirical relationship to combat effectiveness.

CHAPTER 2

STUDY APPROACH AND METHODOLOGY

2-1. PURPOSE. This chapter outlines the approach and methodology for achieving the objectives stated in Chapter 1. This involves describing the nature of the FER user's survey, the conceptual definitions of FER and force ratio, the nature of the ACSDB, and the analytic approach, including the types of relationships, between computed measures, which are examined.

2-2. USER SURVEY. The user survey consisted of informal interviews with simulation model users at CAA. The associated simulation models are EAGLE, the COSAGE, and the CEM IX. Each user was asked for specific rules that they most typically use to compute FERs from their simulation results. Based on the results of these interviews, a cross-reference table was constructed showing similarities and differences among users in the elements and attributes used in their basic FER computations.

2-3. CONCEPTUAL DEFINITION OF FER. Let RED and BLUE denote two opposing forces in conflict, against each other, during a time period i . The conceptual definition of *FER favoring side BLUE in period i* is then defined by the following:

$$FER(i) = (\Delta LOSSES_{RED(i)} / ONHAND_{RED(i)}) / (\Delta LOSSES_{BLUE(i)} / ONHAND_{BLUE(i)}) \quad \text{Eq 2-1}$$

where:

$\Delta LOSSES_{RED(i)}$ = total losses of specified RED items during period i

$ONHAND_{RED(i)}$ = total onhand RED items in period i

$\Delta LOSSES_{BLUE(i)}$ = total losses of specified BLUE items during period i

$ONHAND_{BLUE(i)}$ = total onhand BLUE items in period i

where the items are either personnel or weapon systems which are summed without numerical weights or dimensions (e.g., M tanks lost + N mortars lost = $[M+N]$ items lost).

The ambiguity (or flexibility) in computation arises from:

- a. Whether personnel or weapon systems are used to compute FER.
- b. Which weapon system types are included in the FER calculations.
- c. Which types of losses are included in system-based FER calculations (e.g., only destroyed items or the sum of destroyed and damaged items.)

- d. Which types of casualties are included in personnel-based FER calculations.
- e. The time within the period at which the onhand and/or loss status is registered for use in the FER calculations.

Most CAA users use the conceptual definition of FER defined above. However, one team of CEM users at CAA also uses total authorized items instead of total onhand items in the above definition. This variant of FER has different qualitative and quantitative implications from the conventional definition of Equation 2-1, but is not treated in this paper. Future work concerning use of FER should also consider this variant, since it is sometimes operationally applied.

2-4. CONCEPTUAL DEFINITION OF FORCE RATIO. With opposing RED and BLUE forces defined as above, the force ratio *in favor of RED* in time period *i* is defined as:

$$\text{Force Ratio}(i) = (\text{ONHAND}_{\text{RED}(i)} / \text{ONHAND}_{\text{BLUE}(i)}) \quad \text{Eq 2-2}$$

where:

ONHAND_{RED(i)} = total onhand RED items at start period *i*

ONHAND_{BLUE(i)} = total onhand BLUE items at start of period *i*

where the items are either personnel or weapon systems.

Ambiguity (or flexibility) in computation arises from:

- a. Whether personnel or weapon systems are used to compute force ratio.
- b. Which weapon system types are included in the calculations.

2-5. STRUCTURE OF THE ACSDB

a. Historical Development of the ACSDB. In September 1987, the Historical Evaluation and Research Organization (HERO) was issued a contract to construct a comprehensive history data base describing the WWII Ardennes 1944-45 campaign data base in sufficient detail for simulation. Historical data from forces in the Ardennes Campaign were collected, under contract, and were reformatted into a computerized data base formatted in DBASE IV. The contractor used primary and secondary sources on file at libraries and archives in the United States, Great Britain, and the Federal Republic of Germany. This data base, designated as the Ardennes Campaign Simulation Data Base, was completed in December 1989 by Data Memory Systems, Incorporated.

b. Unit Status Information in the ACSDB. The ACSDB tracks data for divisions and for independent/separate brigade-size German, US, and United Kingdom (UK) units during the

1944-45 Ardennes Campaign on a daily basis. Unit status information provided, for each unit on each day, includes:

(1) Personnel and weapon system inventories and losses. System loss types include destroyed, abandoned, and damaged. Personnel casualty categories include KIA, CMIA, WIA, and disease/nonbattle injury (DNBI).

(2) Number of replacements for weapon systems and for personnel.

(3) Number of vehicles in repair and number of vehicles returning from repair.

(4) Locations of subelements of each unit.

(5) Amounts of ammunition, fuel, and other supplies onhand, received, and consumed during the course of the campaign.

2-6. ANALYTIC APPROACH TO FER ANALYSIS. Figure 2-1 describes the components of FER analysis performed in the ARFERR effort. The process involves using a combination of attributes to define, from the ACSDB, associated sets of historical FER values as well as sets of force ratio values for forces and periods during the historical campaign. Relationships between the computed historical FERs and force ratios are then examined. Relationships between these historical FERs and historical combat effectiveness, and between force ratio and combat effectiveness, are also examined, where combat effectiveness is expressed as FEBA movement.

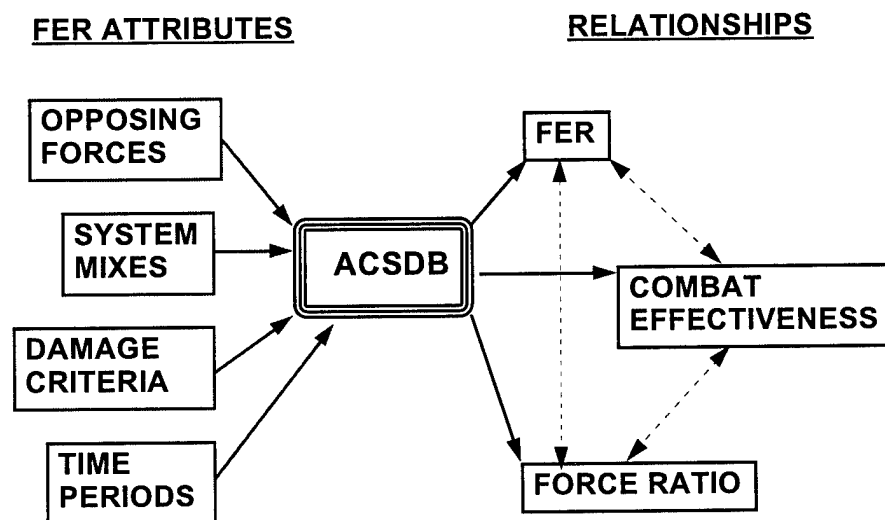


Figure 2-1. FER Analysis Using the ACSDB

a. Generation of FER Cases Using the ACSDB. The basic idea is to first apply the single conceptual definition of FER, favoring the US/UK side, by using the ACSDB data to compute a theater FER, and an ARCAS "bulge" FER, for different cases at 4-day intervals during the 32-

day campaign. Each FER case is characterized by the types of item assets and item losses used in the FER computation. A theater FER is calculated based on the entire theater battle represented in the ARCAS scenario. A theater FER for a time period is based on assets and losses from all of the ACSDB line units committed to the Ardennes conflict during each 4-day period in the ARCAS scenario. An ARCAS bulge FER is calculated based on only those committed line units in the ACSDB which comprise the historical “bulge” in the ARCAS theater. The units in the ARCAS bulge consist chiefly of the German units of the 5th Panzer Army and the US/UK units opposing them. These two forces/battles correspond exactly to their definition in the ARCAS study. The units used in the corresponding FER computations are listed in Appendix D. Each theater FER and each bulge FER are calculated for the 10 cases defined by the attributes listed in Table 2-1.

Table 2-1. FER Case Definitions/Attributes

Type FER	Damage/casualty criterion	Weapon system mix
Personnel	Total casualties	Not applicable
	Combat casualties (KIA, WIA, CMIA)	Not applicable
System	Destroyed or abandoned	Tanks, APCs, AT/Ms, arty (all major systems)
	Destroyed or abandoned	Tanks, AT/Ms, arty (no APCs)
	Destroyed or abandoned	Tanks, AT wpns, arty (no APCs, mortars)
	Destroyed or abandoned	Tanks only
	Destroyed, abandoned, or damaged	Tanks, APCs, AT/Ms, arty (all major systems)
	Destroyed, abandoned, or damaged	Tanks, AT/Ms, arty (no APCs)
	Destroyed, abandoned, or damaged	Tanks, AT wpns, arty (no APCs, mortars)
	Destroyed, abandoned, or damaged	Tanks only

(1) Personnel FER Cases. A uniquely defined personnel theater FER and a personnel bulge FER, using only personnel assets and losses from the ACSDB, are computed for the two cases with losses based on each of the following casualty criteria:

(a) Total casualties (includes KIA, CMIA, WIA, and DNBI)

(b) Combat casualties (includes only KIA, CMIA, and WIA)

(2) System FER Cases. A uniquely defined (weapon) system theater FER and a system bulge FER, using only weapon system data from the ACSDB, are computed for the eight cases defined by a combination of settings from the following case attributes:

(a) System Mix. A system FER is computed using each of the five following groups of weapon system types (as onhand and losses) in the FER calculation.

1. Tanks, armored personnel carriers (APCs), vehicular/towed antitank systems and mortars (denoted as AT/Ms herein), and artillery. This group is also denoted as "all major systems" within charted results in this paper.

2. Tanks, vehicular/towed antitank weapons and mortars, and artillery. This group is also denoted as "no APCs" in charts within this paper.

3. Tanks, vehicular/towed antitank weapons, and artillery. This group is also denoted as "no APCs, mortars" in charts within this paper.

4. Tanks only.

Assignment of an ACSDB weapon to a system type in the above mixes corresponds exactly to the weapon type classification used in the ACSDB.

(b) System Damage Criteria. Each system FER is computed using each of the following damage criteria to characterize losses in the calculation:

1. Destroyed or abandoned.

2. Destroyed or abandoned or damaged.

A theater FER and a bulge FER are computed for each 4-day interval in the campaign, resulting in a total sample of 16 FER values for each of the 10 FER cases.

b. Generation of Force Ratio Cases Using the ACSDB. In addition to the above FERs, theater force ratios and bulge force ratios, in favor of the German side, are also computed from the ACSDB at 4-day intervals during the 32-day campaign. A theater force ratio is calculated based on the entire theater battle represented in the ARCAS scenario, and a bulge force ratio is based on only those committed line units in the ACSDB which comprise the historical "bulge" in the ARCAS theater. Theater and bulge force ratios are computed for each of the five force ratio cases shown in Table 2-2. Personnel force ratios are computed by using only the ACSDB onhand personnel in the force ratio computation. System force ratios are computed using only ACSDB onhand weapon systems in the force ratio computation. The system force ratios are defined for each of the four weapon mixes used in the FER cases. A theater force ratio and a bulge force ratio are computed for each 4-day interval in the campaign, resulting in a total sample of 16 force ratio values for each force ratio case.

c. Plotting the Relationship Between FER Case and Force Ratio. If damage/casualty criteria are ignored, each FER case corresponds to a unique force ratio case defined in Table 2-2. The 16 FERs for each FER case are plotted against the 16 force ratios for the corresponding force ratio case. The associated case plot is labeled herein with the FER case descriptors and has one axis labeled "FER" and the other axis labeled "force ratio."

Table 2-2. Force Ratio Case Definitions/Attributes

Force ratio	Weapon system mix
Personnel	Not applicable
System	Tanks, APCs, AT/Ms, arty (all major systems) Tanks, AT/Ms, arty (no APCs) Tanks, AT wpns, arty (no APCs, mortars) Tanks only

d. Regression Fit Between FER Case and Force Ratio. For each case plot of FER versus force ratio, a regression trendline with a specified equation form which expresses force ratio in terms of FER is fitted to the plotted data using a statistical least squares technique. In addition to the formula of the fitting equation, the fitting process computes a value for the coefficient of determination, denoted by R^2 , which evaluates the reliability of the trend line by indicating how well the fitting equation explains the relationship between the two plotted factors. The R^2 value is sometimes interpreted as the fraction of variation (in the relationship between the plotted variables) that is explained by the fit. A value of $R^2 = 1.00$ corresponds to a perfect fit in which the fitting equation is coincident with the plotted data. An equation of the following exponential form is fitted to each FER-force ratio case plot.

$$\text{FER} = A[e^{B(\text{FR})}]$$

Eq 2-3

where:

FR denotes force ratio

FER denotes fractional exchange ratio

A and B are constants

e denotes the base of the natural logarithm

The exponential form equation produced better fits than a linear or logarithmic form. Therefore, no other regression results are shown.

e. Ranking of Regression Results. After a statistical fit using the above exponential form is done for each FER-force ratio case plot, the results are ranked in the order of decreasing value of resulting R^2 , which is essentially a ranking in order of "goodness of fit." The best fit corresponds to the case in which the fitting equation best "predicts" force ratio from FER *for the specific sample of points plotted*. The smallness of the statistical sample (16 points) limits the usefulness of either the ranking, or the value of R^2 , as an indicator of the predictive value of the plotted relationship beyond the specific sample used in the fit. These rankings, and any observations derived from them, should therefore be treated as suggestive only.

f. Relating FER and Force Ratio to Combat Effectiveness. In addition to analyzing the relationship between historical FER and force ratio, the ARFERR analysis also examines the relationship between historical FER and combat effectiveness, and between historical force ratio and combat effectiveness, where the measure of historical combat effectiveness is the average historical FEBA movement during each time period for the case examined. The data on historical FEBA movement are those generated during the ARCAS Study. The FEBA movement measures used are the following:

(1) Average historical FEBA movement over the ARCAS theater during each 4-day interval of the Ardennes Campaign.

(2) Average historical FEBA movement over the "bulge" in the ARCAS theater during each 4-day interval of the Ardennes Campaign.

Instead of examining the direct relationship between FER and FEBA movement, FER is first transformed into an approximation of another measure, denoted as the advantage factor, which is a statistic that has been shown, under certain conditions, to be one of the best measures of the combat effectiveness of a force, relative to an opposing force. The specific transformation of FER into the advantage factor uses the following equation:

$$ADV = -.5[\ln(FER)] \quad \text{Eq 2-4}$$

where:

ADV denotes the advantage factor favoring the German side

FER denotes fractional exchange ratio favoring the US/UK side

Ln denotes the natural logarithm function

As noted below, the relationship expressed in Equation 2-4 is an approximation which is both well-known and supported by both theoretical and empirical evidence for personnel casualties in large forces. Using the above transformation, the relationship between FER and FEBA movement is expressed in ARFERR results as the plotted and fitted relationship between the ADV associated with a FER to the ARCAS FEBA movement for the force and time period associated with the FER case.

g. Definition and Nature of the Advantage Factor. The advantage factor (Refs 4, 5), as defined below, is well-known as the major parameter influencing the course of battle for Lanchester-like attrition. It is based on a set of purely mathematical propositions rigorously derived from the equations underlying Lanchester combat.

$$\text{ADV}(\text{RED}) = .5\text{Ln}[(1 - a^2)/(1 - b^2)] \quad \text{Eq 2-5}$$

where:

ADV(RED) denotes the advantage factor favoring side RED

a = final BLUE strength/BLUE initial strength

b = final RED strength/RED initial strength

According to Dr. Robert Helmbold, the advantage factor, *when computed using the entire personnel strength of a force*, is “demonstrably the best currently available measure of the overall combat effectiveness of an entire force vis-à-vis its opponent’s entire force.” The theoretical basis of the advantage factor is based only on personnel strengths and casualties for the forces opposed.

It is also known that:

$$\text{ADV}(\text{BLUE}) = -\text{ADV}(\text{RED}) \quad \text{Eq 2-6}$$

and that the following approximate relationship will generally exist between FER and ADV

$$\text{ADV}(\text{RED}) = .5\text{Ln}(\text{FER favoring side RED}) \quad \text{Eq 2-7}$$

where Ln denotes the natural logarithm.

h. Qualification on Interpretation of the ADV Factor in ARFERR. Only the personnel ADV values for the personnel FER case plots and the system ADV values for the system FER case plots for the FER cases with “all major systems” will include all (personnel or major system) elements of each force. The other system ADV values are really “partial ADVs” because the associated system mixes omit some major system types. There is no theoretical justification for the worth of a partial ADV as a measure of combat effectiveness since the theoretical basis of the advantage factor requires a complete inclusion of all elements of a force. Additionally, the theory of the ADV has been justified and empirically demonstrated only for ADV defined in terms of personnel elements of a force. Therefore, no reliability, relative to predicting combat effectiveness, can be theoretically ascribed to ADV values which include weapon systems, or which only include some components of each force.

i. Relating the FER-derived ADV Factor to FEBA Movement. Once the FER values for each FER case are transformed (using Eq 2-4) into ADV factors, the resulting 16 ADV values are plotted against the 16 historical FEBA movement values comprising the FEBA movement in each 4-day period over the ARCAS theater and over the ARCAS "bulge." These case plots for ADV versus FEBA movement have one axis labeled "ADV" and the other axis labeled "km advance." The case plots for ADV vs FEBA movement are then analyzed exactly analogous to the way in which the FER-force ratio case plots are analyzed, except that only linear trend line fits are applied. From Equation 2-4, it follows that each linear trend line equation for ADV versus FEBA movement in this paper is equivalent to a logarithmic trend line for FER versus FEBA movement.

j. Relating Force Ratio to FEBA Movement. The force ratio values plotted for each case are plotted against the same historical FEBA movement values used to relate ADV factor to FEBA movement. These case plots for force ratio vs FEBA movement are then analyzed exactly analogous to the way in which the FER-force ratio case plots are analyzed, except that only linear and logarithmic trend line fits are applied.

CHAPTER 3

USER SURVEY

3-1. INTRODUCTION. This chapter describes the results of a survey of simulation model users at CAA on the way that they define, and most typically compute, FER, and how they interpret the need for, and value of, a standardized FER computational method. The survey was done through informal interviews. The simulation models used by the interviewees are:

a. EAGLE. EAGLE is a deterministic model of combat up to corps level. EAGLE is used to assess comparative weapon system effectiveness in support of the Value Added Analysis studies at CAA. The Value Added Analysis studies provide guidelines for program issue tradeoffs during planning of the overall Army budget.

b. Concepts Evaluation Model (CEM). CEM is a deterministic model of theater combat which has been used extensively at CAA to support both capability assessment and force structuring. The CEM interviews were conducted with representatives from two different analytic teams at CAA. One team primarily analyzes contingencies in Northeast Asia (NEA), while the other team models contingencies in other parts of the world. Responses from the two teams are separately described in the interview results.

c. Combat Sample Generator (COSAGE). COSAGE is a stochastic model of small unit combat (usually brigade and division level) which functions at CAA as a preprocessor to CEM. Ammunition consumption and attrition for representative division battles of a CEM scenario are first modeled in COSAGE. Later, during CEM execution, COSAGE results of those representative battles (denoted as combat samples) are used in an interpolation algorithm, denoted as the Attrition Calibration (ATCAL) process, to generate attrition and ammunition consumption results for simulated CEM combat.

Except for CEM, the users' labels are equated to the names of the associated models. In the case of CEM, the users are labeled CEM/NEA and CEM/OTHER according to whether the responses are from the team analyzing NEA contingencies or other contingencies.

3-2. USER APPLICATION OF FER ATTRIBUTES. Most CAA users use the conceptual definition of FER defined in Chapter 2, which is the only type of FER computation treated in this paper. Users differ, however, in the way they define and apply specific components and attributes to the FER calculation. The following is a summary of the ways that specific attributes of the basic FER computation method are defined and applied by each interviewed user. The CEM/OTHER entry for each attribute is stated as "varies" in all cases because the user response was that they had no typical way for computing FER, and that user treatment of the listed attributes was completely flexible and depended on specific study requirements. The other user responses represent their most typical treatment of the listed attributes in FER calculations. These comparative summary responses are summarized in Table 3-1.

Table 3-1. Application of FER Attributes by Users of CAA Simulations

FER attribute	EAGLE users	COSAGE users	CEM/NEA users	CEM/OTHER users
Period	Battle duration (appr. 3-6 days)	2 days	4 days	Varies
System types included in losses (and onhand)	All tanks, vehicular ATWs, crew-mounted TOW, attack helicopters, air defense wpns (except man-carried), mortars larger than 120mm, artillery	All tanks, vehicular ATWs, attack helicopters, AD weapons, artillery (TACAIR)	All tanks, vehicular ATWs, attack helicopters, mortars larger than 100mm, artillery	Varies
Excluded Army wpn systems	Man-carried weapons (except for STINGER and TOW), mortars of 120mm or less, APCs w/o AT capability	Man-carried weapons (except for TOW), all mortars, APCs w/o AT capability	Man-carried weapons (including TOW), all AD weapons, mortars of 100mm or less, APCs w/o AT capability	Varies
TACAIR system treatment	Ignored in onhand, but kills by TACAIR are included in losses	A "non-TACAIR FER" excludes TACAIR and kills by TACAIR, a "TACAIR FER" includes onhand/lost TACAIR (nr AC) and kills by TACAIR	None, but TACAIR kills are included in losses	Varies
Types of losses included	Destroyed and damaged in combat	Destroyed and damaged in combat	Destroyed (permanent combat losses)	Varies
Types of losses excluded	None (maintenance failures are not currently modeled)	Kills by TACAIR are excluded from the non-TACAIR FER	Temporary losses	Varies
Time of onhand status assessment	Beginning of first day of period treated	Beginning of first day of period treated	End of last day of period treated.	Varies
Treatment of maintenance returns in period	Not currently considered. When log module is functional, returns will be added to onhand for period	Not considered (maintenance and repair are not modeled)	Not applicable to permanent losses	Varies

a. Time Period. The duration of the battle period over which the losses are summed for use in the FER calculation.

(1) **EAGLE.** This depends on the specific scenario, but is typically 3-6 days for applications.

(2) **COSAGE.** A FER is computed at 2-day intervals during the simulated battle.

(3) **CEM/NEA.** A FER is computed at 4-day intervals during the simulated battle.

(4) **CEM/OTHER.** Varies.

b. System Types Included in FER Computation

(1) **EAGLE.** All of the following: tank systems, vehicular antitank weapons, crew-mounted tube-launched, optically tracked, wire-guided (TOW) missiles, attack helicopters, air defense weapons that are not man-carried (except STINGER), mortars with diameter greater than 120mm, and artillery. Mortars with diameter greater than 120mm are included because the Conventional Forces in Europe (CFE) Treaty equates mortars of these sizes with artillery.

(2) **COSAGE.** All of the following: tank systems, vehicular antitank weapons, crew-mounted TOWs, attack helicopters, air defense weapons, and artillery. Aircraft performing close air support may be optionally included.

(3) **CEM/NEA.** All of the following: tank systems, vehicular antitank weapons, attack helicopters, mortars with diameter greater than 100mm, and artillery.

(4) **CEM/OTHER.** Varies.

c. System Types Excluded in FER Computation. The types of land warfare systems excluded in the FER computation are:

(1) **EAGLE.** All man-carried weapons (except for STINGER and TOW), mortars with diameter of 120mm or less, and light armor/APCs without antitank capability.

(2) **COSAGE.** Man-carried weapons (except for TOW), mortars, and light armor/APCs without antitank capability.

(3) **CEM/NEA.** Man-carried weapons (including TOW), air defense weapons, mortars with diameter 100mm or less, and light armor/APCs without antitank capability.

(4) **CEM/OTHER.** Varies.

d. Treatment of Tactical Air. The types of tactical aircraft included in the FER computation.

(1) **EAGLE.** No tactical assets are counted in the onhand, but kills by tactical air (TACAIR) are included in losses.

(2) **COSAGE.** Two types of FER can be computed. A non-TACAIR FER excludes onhand and lost TACAIR assets from being counted, and also excludes kills by TACAIR from being counted in losses. An optional TACAIR FER includes both onhand and lost TACAIR assets, expressed as number of aircraft performing close air support, and also includes kills by TACAIR in losses.

(3) **CEM/NEA.** There is no inclusion of TACAIR assets in onhand, and there is no exclusion of TACAIR kills from losses used in the FER.

(4) **CEM/OTHER.** Varies.

e. Types of Losses Included in FER Computation. System losses may be either permanent (destroyed or abandoned) or temporary (damaged and repairable).

(1) **EAGLE.** Systems that are destroyed as well as those that are damaged in combat are included.

(2) **COSAGE.** Systems that are destroyed as well as those that are damaged in combat are included.

(3) **CEM/NEA.** Only permanent losses (destroyed or abandoned systems) are included.

(4) **CEM/OTHER.** Varies.

f. Types of Losses Excluded in FER Computation

(1) **EAGLE.** None (maintenance failures are not currently modeled).

(2) **COSAGE.** Kills by TACAIR are excluded from the non-TACAIR FER.

(3) **CEM/NEA.** Temporary losses (damaged systems).

(4) **CEM/OTHER.** Varies.

g. Time of Onhand Status Assessment. This refers to the point in time at which the “snapshot” of assessed onhand assets is made for the FER computation.

(1) **EAGLE.** The onhand status is assessed at the beginning of the first day of the time period.

(2) **COSAGE.** The onhand status is assessed at the beginning of the first day of the time period.

(3) **CEM/NEA.** The onhand status is assessed at the end of the last day of the time period.

(4) **CEM/OTHER.** Can vary.

This survey, and paper, do not treat the variant FER computation, sometimes used at CAA, in which authorized items are used instead of onhand items.

h. Treatment of Maintenance Returns

(1) **EAGLE.** Maintenance is not currently modeled. After a logistics module is implemented in EAGLE, maintenance returns will be added to the onhand assets for the period.

(2) **COSAGE.** Maintenance is not currently modeled.

(3) **CEM/NEA.** Maintenance returns do not apply to permanent combat losses.

(4) **CEM/OTHER.** Varies.

3-3. TREATMENT OF DIFFERENCES IN WEAPON EFFECTIVENESS. The existence of significant differences in firepower and lethality among weapon types raises a question of whether a system's presence in the FER computation should be numerically weighted in some proportion to its lethality. Other than the numerical weights (1 or 0) implicitly used in including and excluding systems in the computation, no numerical system weights are applied by any user in FER calculations. In general, model users at CAA attempt to include, in the FER computation, only artillery systems and those weapon systems that have a potential for killing armor. The user justifications for exclusion of light/man-carried antitank systems include the following rationales:

a. Man-carried antitank weapons (ATWs) (e.g., DRAGON/JAVELIN) are usually not included because, although effective at 1,500-2,000m, they are not likely to be used, since other systems will engage enemy tanks before the man-carried systems can be used effectively. However, in campaigns on very dense/cluttered terrain, masking may cause the vehicular systems to have their major engagements at closer ranges, in which case the man-carried ATWs may be used much more. In those cases, we should probably include them (EAGLE user).

b. A man-carried ATW (e.g., DRAGON) is generally only used in defense combat posture and even then is not a big killer. Its use is negligible in other postures, such as attack, or static (COSAGE user).

Since no user applies differential numeric weighting of weapon systems (other than inclusion or exclusion) to FER computation, numeric weighting of weapons is ignored in all analyses described herein.

3-4. USER INTERPRETATION OF THE VALUE OF A STANDARDIZED FER COMPUTATION. All CAA model users regard the FER as one of many indicators of what happened in simulated battle. FER is considered to be no more important than other outcome MOEs. A CEM user commented that he believed that the FER was only used for dissemination at internal reviews and briefings within CAA, and that few, if any, agencies outside of CAA use it in external reports. All users indicated that they use FER primarily for internal review and analysis of battle results. The following points were made by users about the need for, and usefulness of, a standardized computational methodology for FER:

a. All users believe that the most important aspect of FER use at CAA is for it to be computed in the same way from scenario to scenario, and from year to year, in results generated by that user's model. Each user (except CEM/OTHER) already has a standard method of computing a basic FER and regards retention of that method as necessary to ensure consistency and reliability in FER comparisons over scenarios. Standardization would likely require a

redefinition of a user's basic FER, but the redefined standard FER would not be comparable with old-style FERs generated prior to standardization. (Such comparisons would mix scenario-based differences with differences caused by altered inclusion/exclusion criteria for systems.)

b. Since the user models vary in the types and resolution of the combat processes modeled, aspects of FER computation may be more easily or flexibly changed in some models than in others. Often, the attributes which a user includes or excludes in their FER calculation are tailored to, or constrained by, what the user's model can, or can't, generate for use in the FER calculation. For example:

(1) Neither EAGLE nor COSAGE considers maintenance returns because maintenance is not simulated in those models.

(2) COSAGE cannot readily restrict FER calculation to permanent losses because it cannot separate permanent losses from temporary losses.

Overall, CAA model users acknowledge that, if FERs are used, each user should have a standard computational method to define the FERs from their scenarios. Users were indifferent to imposition of a standardized FER computation within all of CAA but believed that use of their current (user-specific) FER would have to continue anyway so that comparisons could be done with FERs from past years. No user believed that use of FER was absolutely necessary as an MOE of modeled combat outcome.

CHAPTER 4

FER RELATIONSHIP TO FORCE RATIO

4-1. INTRODUCTION. This chapter presents the results from using the ACSDB to analyze the relationship between FER and force ratio with the methodology described in Chapter 2. The FERs derived from the ACSDB are all computed using the same conceptual FER definition (presented in Chapter 2) and are based on the case attributes listed in Table 2-1. All FERs are computed in favor of the US/UK side (using German [losses/onhand] in the numerator)). All force ratios are computed in favor of the German side. This chapter shows the variation in FER values over the different cases and trend line relationships between computed FER and force ratio (FR). FER versus force ratio is plotted for 16 ACSDB battles in each case, and a best-fit exponential-form trend line is statistically fitted to the plotted points. Quantitative measures of goodness of fit are used to assess the nature and strength of relationships between FER and FR and to develop general observations. Results for all cases studied are included in tables shown, but only a selection of graphical results is presented here. Graphical results for all cases analyzed are presented in Appendix E.

4-2. VARIATION IN FER OVER CASES

a. FER Over the ARCAS Theater. Figures 4-1 and 4-2 show the computed theater FER values in the 10 cases studied for each 4-day time period in the Ardennes Campaign. Figure 4-1 is based on FER using losses in terms of damaged, destroyed, and abandoned systems for system FER, and total personnel casualties for personnel FER. Figure 4-2 is based on FER using losses in terms of destroyed and abandoned systems for system FER, and total combat casualties for personnel FER. A theater FER for a time period is based on assets and losses from all of the ACSDB line units committed to the Ardennes conflict during each 4-day period in the ARCAS scenario. Each figure shows personnel FERs for one case and system FERs for four cases. Table 4-1 associates the personnel legends presented in the figures with the complete description of casualty criterion presented in Table 2-1. Table 4-2 associates the system legends presented in the figures with the complete system mix descriptions presented in Table 2-1. These legends will be used extensively in other figures in this chapter and in the appendices.

Table 4-1. Personnel FER Casualty Criteria Used

Legend in figure	Casualty criterion
TOT PERS CAS	Total casualties
PERS CBT CAS	Combat casualties (KIA, WIA, CMIA)

Table 4-2. FER System Mixes Used

Legend in figure	Weapon system mix for case
ALL MAJOR SYSTEMS	Tanks, APCs, AT/Ms, arty
NO APCS	Tanks, AT/Ms, arty
NO APCS, MORTARS	Tanks, AT wpns, arty
TANKS ONLY	Tanks only

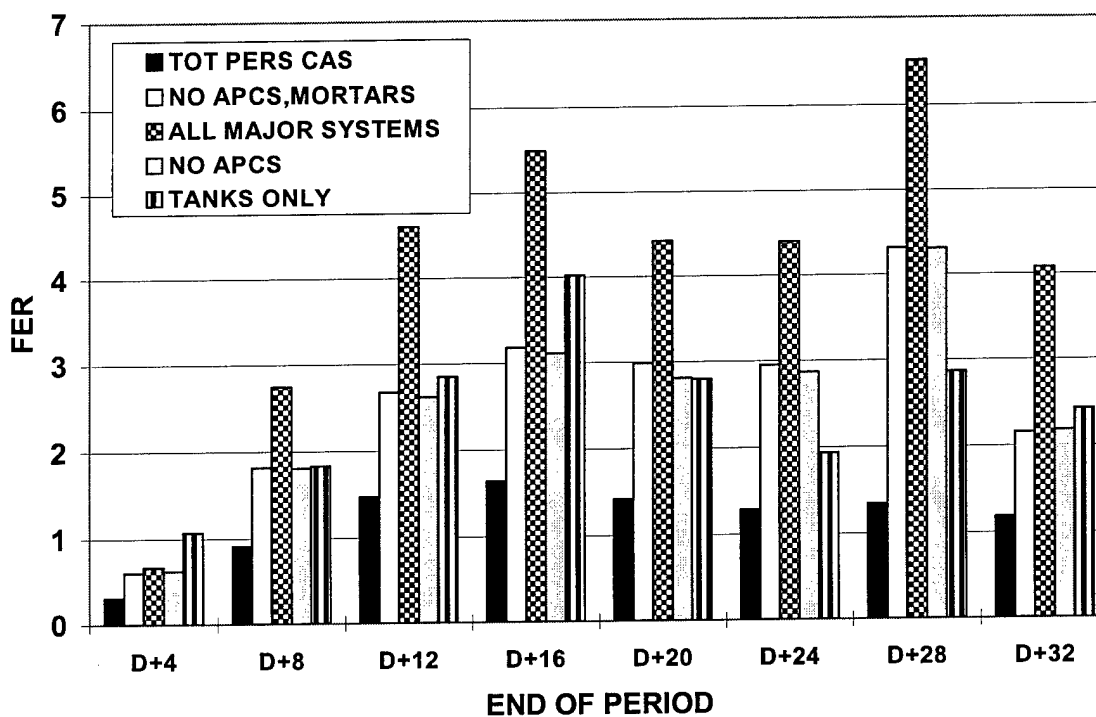


Figure 4-1. ARCAS Theater FERs by 4-day Period Using Losses in Damaged, Destroyed, and Abandoned Systems

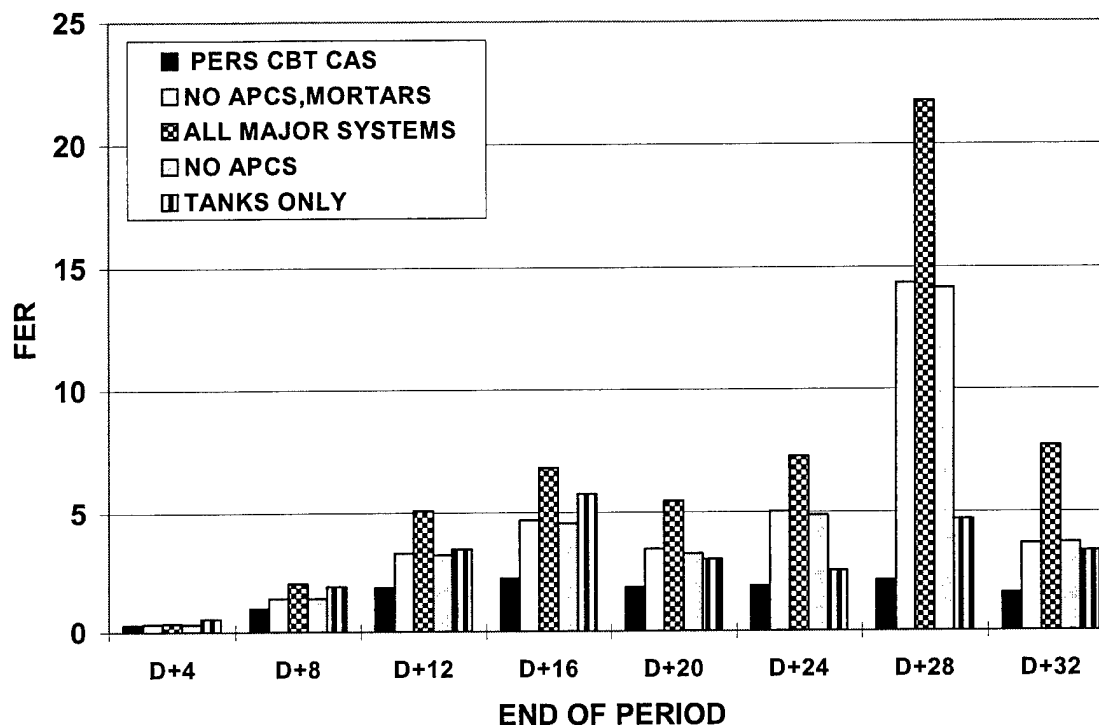


Figure 4-2. ARCAS Theater FERs by 4-day Period Using Losses in Only Destroyed and Abandoned Systems

Since the FERs portrayed here are calculated in favor of the US/UK side, a value less than 1.00 indicates that the US/UK lost a greater portion of their force than the Germans in that time period. Conversely, a FER greater than 1 indicates that the Germans lost a larger fraction of their force than the US/UK. The following observations can be made from the depicted variation.

(1) In general, the system FER based on all major systems consistently produces the largest system FER values for time periods after D+16, the system FER based on [tanks only] produces the lowest system FER values. The personnel FER is consistently less than any system FER in each time period.

(2) The variation in personnel FER over the eight time periods is considerably less than the variation in any system FER over the time periods. Among the system FERs, the FER using all major systems has the most variation over the campaign, while that based on [tanks only] has the least variation.

(3) The system FER values using no APCs are nearly the same as the system FER values using no APCs or mortars.

(4) The personnel FERs are less than 1 only in the first two 4-day periods, while the system FER is less than 1 only in the first 4-day period. This indicates that, in a sense, the Germans were losing a greater fraction of their committed weapon systems than the US/UK after D+4.

(5) The system FERs based on damaged, destroyed, or abandoned systems are generally less than the system FERs based on only destroyed or abandoned systems for time periods after D+12. However, the two types of system FER are nearly equal in the second 4-day time period, while that based on damaged, destroyed, or abandoned systems is larger in the first 4-day time period. The difference is especially large in the 4-day period ending in D+28 when the FER based on only destroyed or abandoned systems is about four times larger (15 -22 versus 4 - 6.5).

(6) The FER based on combat casualties is almost double the FER based on total casualties in time periods after D+8 (a FER of approximately 2 versus a FER of around 1).

b. FER Over the ARCAS Bulge. Figures 4-3 and 4-4 show the computed FER values over the ARCAS bulge in the 10 cases studied for each 4-day time period in the Ardennes Campaign. Figure 4-3 is based on FER using losses in terms of damaged, destroyed, and abandoned systems for system FER and total personnel casualties for personnel FER. Figure 4-4 is based on FER using losses in terms of destroyed and abandoned systems for system FER and total combat casualties for personnel FER. A bulge FER is calculated based on only those committed line units in the ACSDB which comprise the historical "bulge" in the ARCAS theater. Each figure shows personnel FERs for one case and system FERs for four cases. Bulge FER values are similar to the corresponding values for theater FERs and the observations stated for theater FERs also generally apply to the bulge FERs.

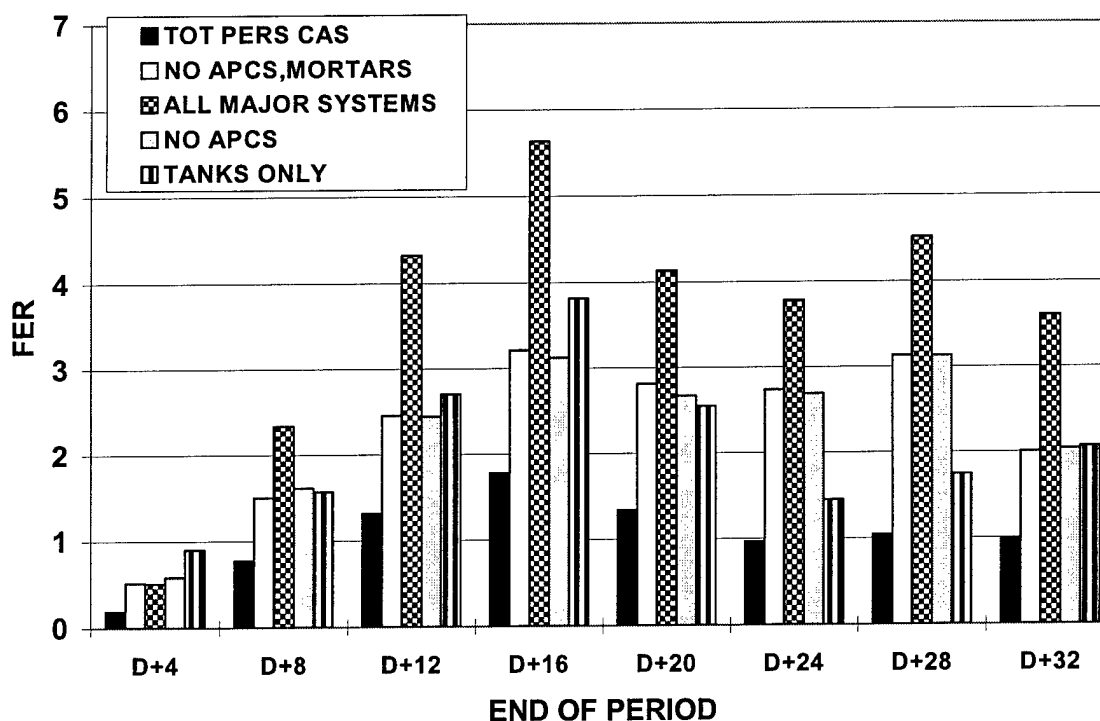


Figure 4-3. ARCAS Bulge FERs by 4-day Period Using Losses in Damaged, Destroyed, and Abandoned Systems

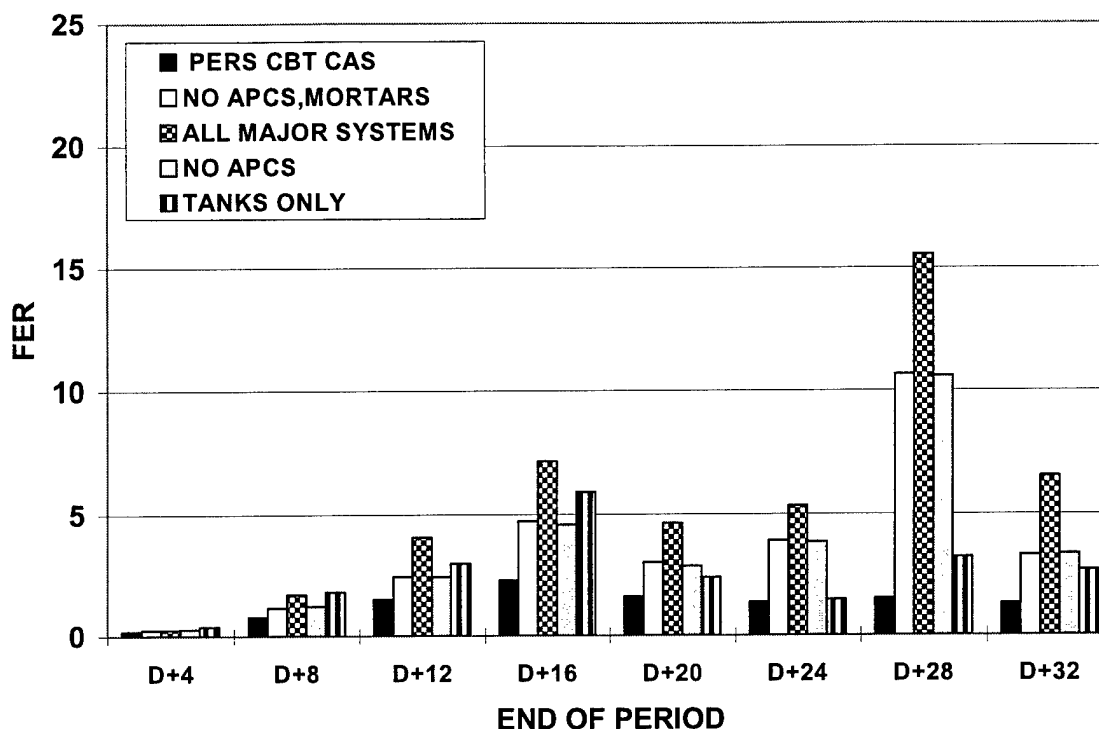


Figure 4-4. ARCAS Bulge FERs by 4-day Period Using Losses in Only Destroyed and Abandoned Systems

4-3. EQUATION FORM USED TO RELATE FER TO FORCE RATIO. The force ratio, defined in Chapter 2, is a computational component of FER. The FER may be regarded as the ratio of two ratios. The FER in favor of side BLUE is defined as CR/FR, where CR is the casualty ratio defined as [RED losses/BLUE losses in the period], and FR is force ratio in favor of RED, defined as [RED onhand/BLUE onhand at the start of the period]. Previous undocumented analyses at CAA have reportedly indicated that force ratio and FER are approximately related by an exponential-form function with equation:

$$\text{FER} = A[e^{B(\text{FR})}]$$

Eq 4-1

Fits to linear-form, power-form and logarithmic-form equations were also examined, but the exponential-form equation yielded trend lines with the best fits in all cases. Therefore, only results for the exponential form are presented and discussed herein.

4-4. ASSESSMENT OF EXPONENTIAL-FORM TREND LINE RELATIONSHIP BETWEEN FER AND FORCE RATIO.

A regression trend line, with the exponential-form expressed in Equation 4-1, was fitted to each of a set of 10 scatter plot cases of FER versus force ratio. Each scatter plot case consists of a set of 16 FERs plotted against 16 force ratios, where each plotted point is an ordered pair (FR, FER) where the FER and FR are computed from the ACSDB, with FER in favor of the US/UK side, and FR in favor of the German side, for a 4-day period during the campaign. Eight of the (FR, FER) pairs/points in each plot are theater FERs/FRs, computed over the full ARCAS theater, and eight are bulge FERs/FRs, computed only over the ARCAS bulge. Each scatter plot case corresponds to one of the 10 FER cases

described in Table 2-1, with all computed FERs and force ratios for the case using the same weapon system mix, and (for FER) damage criterion. The value of R^2 , the coefficient of determination, for each trend line is used to assess the strength of the fitted relationship between FER and force ratio for each case.

a. Trend Line results. Table 4-3 lists the characteristics of the best-fit exponential-form trend line for each of the 10 scatter plot cases. Tabulated case attributes include:

- (1) The associated system mix used in calculating FER and force ratio in the case.
- (2) The associated damage criterion used in computing system FER in the case.
- (3) The value of the coefficient of determination, R^2 , associated with the exponential-form trend line fit (Equation 4-1) for FER vs force ratio in this case.
- (4) The average percent error in each fitted FER derived from the fitting equation applied to each of the 16 historical force ratios plotted for the case. Percent error is the absolute value of the difference [fitted FER - historical FER] divided by the historical FER. The standard deviation of these error percents is also shown.
- (5) The equation of the exponential-form trend line fit for this case. In the equation, FR denotes force ratio at the start of a (4-day) period, and FER denotes the fractional exchange ratio for the period, based on the case definition.

The cases in Table 4-3 are rank-ordered by descending value of R^2 because the coefficient of determination is a measure of "goodness of fit," with a high value (near 1.00) associated with a strong exponential-form relationship between the plotted FER and FR.

b. Case Differences among Exponential-form Fits

(1) Goodness of Fit Based on Coefficient of Determination. Basing strength of the indicated relationship only on the value of R^2 :

- (a) The best fit between FER and force ratio is for the personnel FER case using combat casualties in FER calculations. In this case, $R^2 = .90$.
- (b) The weakest relationships (worst fits) between FER and force ratio are in the system FER cases using only tanks in FER and FR calculations. The case using only destroyed and abandoned tanks has $R^2 = .71$, while the case using damaged, destroyed, and abandoned tanks has $R^2 = .50$.
- (c) All system FER cases except that for [tanks only] have almost equally good fits, with R^2 values between .86 and .89

Table 4-3. Exponential Trend Line Relationship between Historical FER and Force Ratio in the ACSDB

Systems used in FER and FR	Damage criterion	R ² = coeff of determ	Avg % error /std dev in fitted FER	Equation of fitted line
Personnel	Combat casualties	.90	17 % / 12	FER = 4.36e ^{-1.13(FR)}
Tanks, APCs, AT/Ms, arty	Destroyed or abandoned	.89	30 % / 20	FER = 20.78e ^{-2.93(FR)}
Tanks, APCs, AT/Ms, arty	Damaged , destroyed or abandoned	.89	20 % / 12	FER = 8.83e ^{-1.78(FR)}
Tanks, AT wpns, arty	Damaged , destroyed or abandoned	.89	16 % / 15	FER = 4.53e ^{-1.03(FR)}
Tanks, AT/Ms, arty	Damaged , destroyed or abandoned	.88	15 % / 14	FER = 4.63e ^{-1.08(FR)}
Tanks, AT wpns, arty	Destroyed or abandoned	.86	30 % / 20	FER = 10.43e ^{-1.87(FR)}
Tanks, AT/Ms, arty	Destroyed or abandoned	.86	29 % / 20	FER = 11.43e ^{-2.05(FR)}
Personnel	Total casualties	.84	19 % / 12	FER = 2.77e ^{-0.92(FR)}
Tanks only	Destroyed or abandoned	.71	32 % / 32	FER = 4.19e ^{-1.74(FR)}
Tanks only	Damaged, destroyed or abandoned	.50	25 % / 17	FER = 2.77e ^{-0.82(FR)}

(2) Goodness of Fit Based on Percent Error in Fitted FER. An alternative to use of R² as an indicator of goodness of fit is to use the average percent error in fitted FER relative to the historical FER. Based on the average percent error in fitted FER:

(a) The best fit between FER and force ratio is in the system FER case using tanks, AT/M weapons, and artillery systems with a damage criterion of damaged, destroyed, or abandoned systems.

(b) All system FERs for cases using damaged, destroyed, and abandoned systems had markedly better fits to FR than any based on calculations which included only destroyed and abandoned systems. Personnel FER based on combat casualties yielded only a slightly better fit than that based on total casualties.

Figure 4-5 shows the graphical exponential-form trend line relationship between personnel FER and FR. The depicted cases differ only in damage criterion. In the figure, CBT CAS denotes combat casualties, and TOT CAS denotes total casualties. The figure also shows the values of R² and the fitting equations for the two cases. Figure 4-6 shows the graphical exponential-form trend line relationship between system FER and FR in the two cases using all major systems, which includes the best-fit system FER case based on coefficient of determination. The depicted cases differ only in damage criterion. In the figure, DST denotes destroyed or abandoned, and

DST & DMGD denotes damaged, destroyed, or abandoned. The figure also shows the values of R^2 and the fitting equations for the two cases. Graphs for all cases in Table 4-3 are in Appendix E.

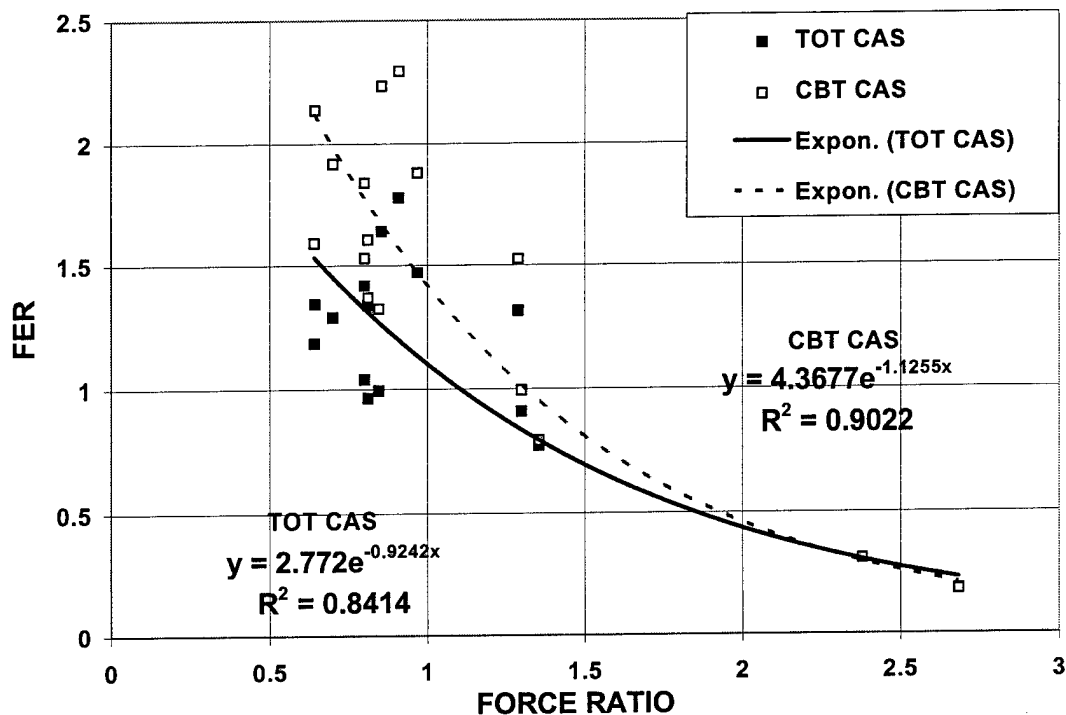


Figure 4-5. Exponential Fit of Personnel FER vs Force Ratio

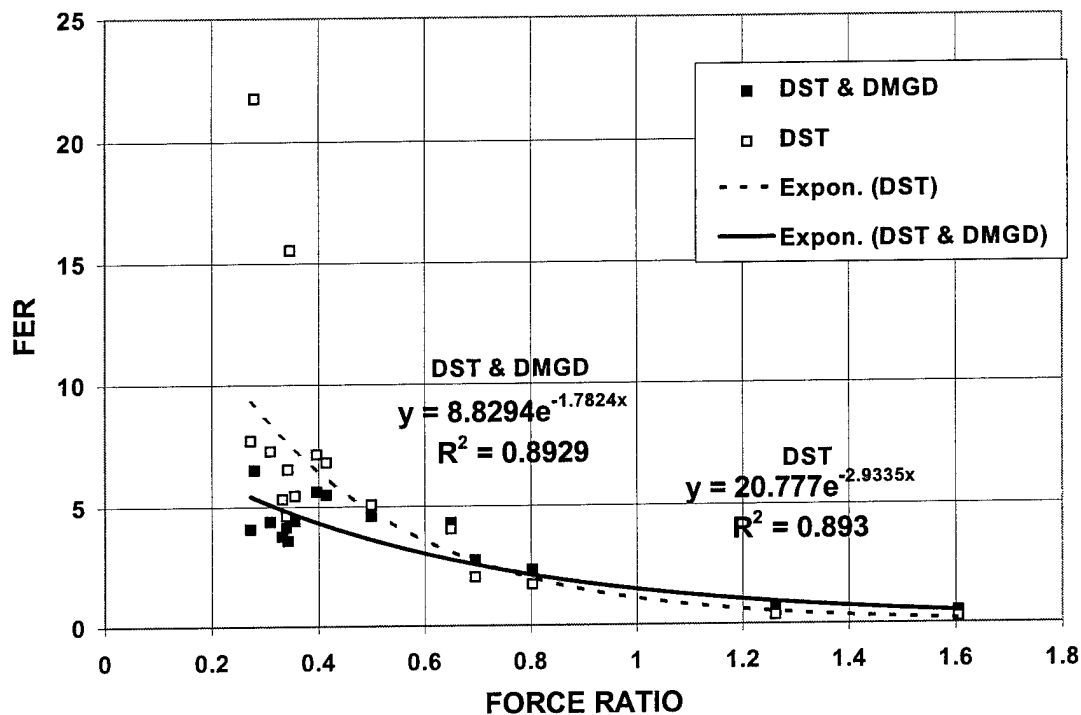


Figure 4-6. Exponential Fit of System FER vs Force Ratio (using all major systems)

c. Fitted versus Historical FER for Exponential-form Fits. A graph, over the course of the campaign, of the absolute difference between the historical FER and the FER derived from the exponential fit to the historical force ratio provides a different perspective from that shown in a scatter plot of FER versus FR. For each case shown in Table 4-3, fitted FER for each 4-day period in the campaign is determined by the exponential fitting equation, using FR equal to the historical force ratio for that period in that case. The complete set of plots of fitted FER versus historical FER, for the cases in Table 4-3, is in Appendix E.

(1) Figure 4-7 shows the exponential-form fitted FER plotted at 4-day intervals against the historical FER, for the best-fit case based on coefficient of determination (personnel FER based on combat casualties). Each fitted FER value for a period plotted in Figure 4-7 is then a y-axis coordinate of the point on the dashed (CBT CAS) line of Figure 4-5 which has an x-axis (FR) value equal to the historical force ratio at the start of that period.

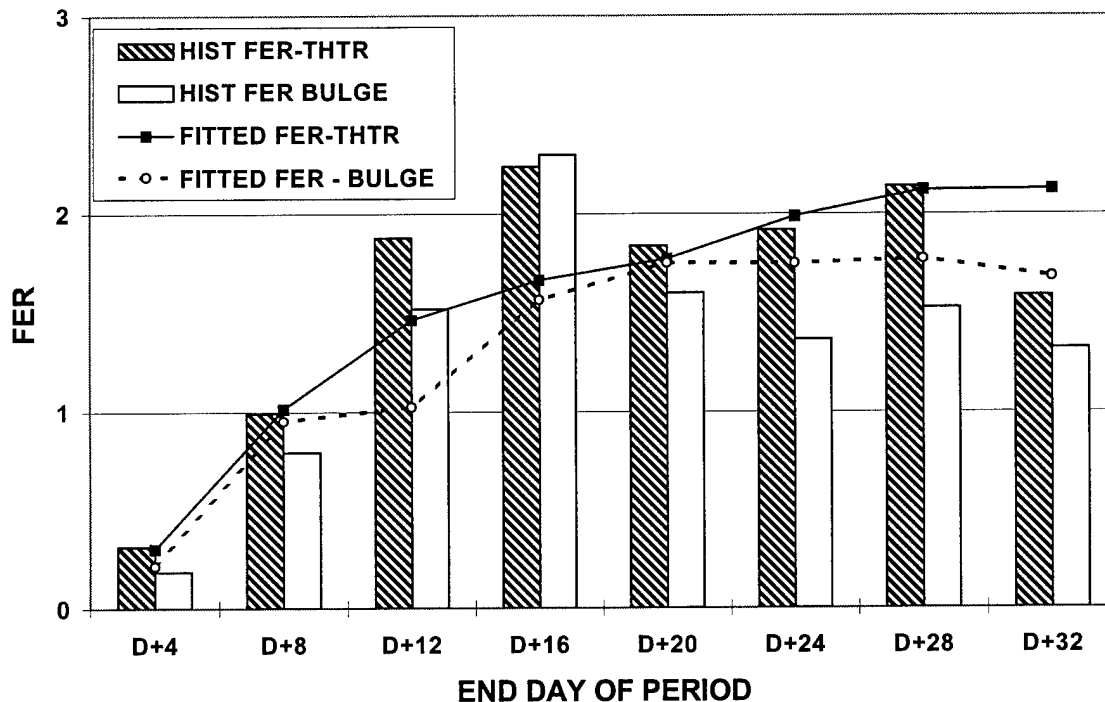


Figure 4-7. Historical Personnel FER vs Exponential Fitted FER (combat casualties)

(2) Figure 4-8 shows the exponential-form fitted system FER plotted at 4-day intervals against the historical FER, for the best-fit case based on coefficient of determination (all major systems with a damage criterion of destroyed or abandoned).

(3) Figure 4-9 shows the exponential-form fitted system FER plotted at 4-day intervals against the historical FER, for the best-fit case based on average percent error in fitted FER (tanks, AT/Ms, and artillery, with a damage criterion of damaged, destroyed or abandoned).

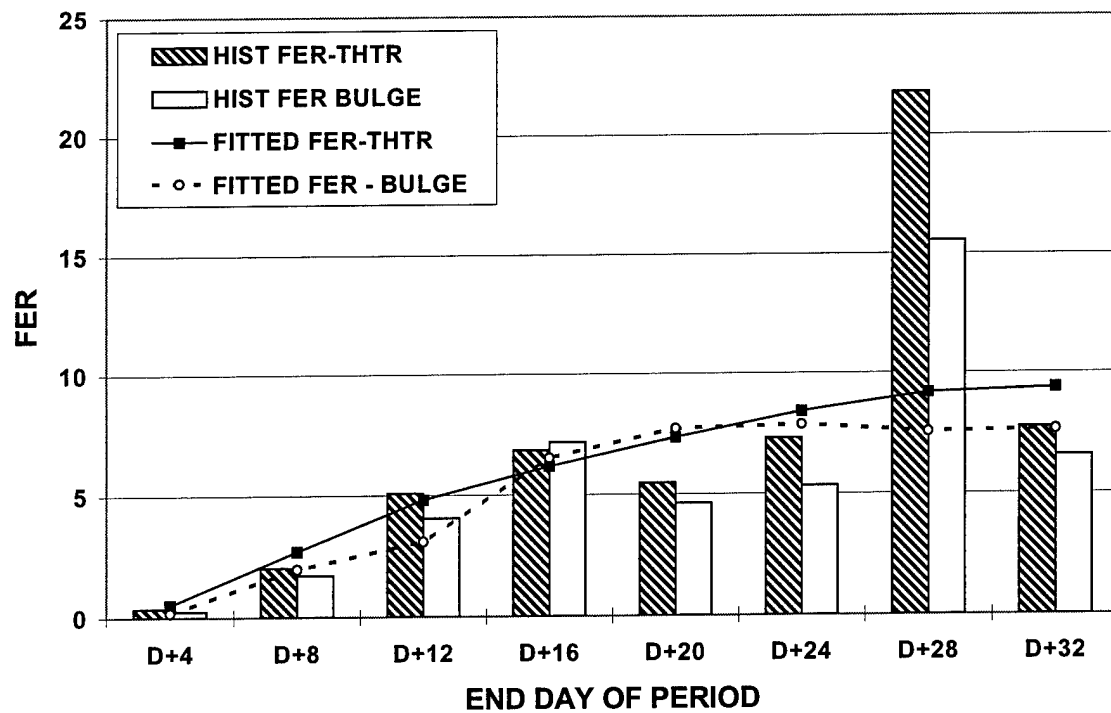


Figure 4-8. Historical FER vs Exponential Fitted FER
(all destroyed and abandoned major systems)

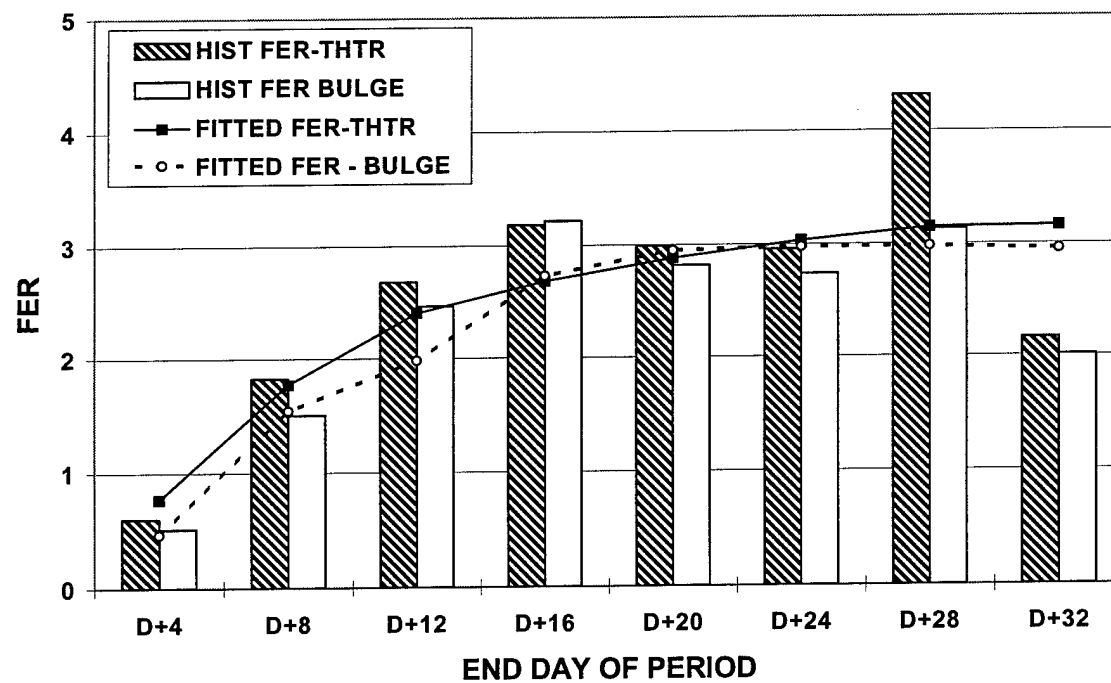


Figure 4-9. Historical FER vs Exponential Fitted FER
(damaged, destroyed, and abandoned tanks, AT/Ms, artillery)

4-5. OBSERVATIONS ON THE RELATIONSHIP BETWEEN FER AND FORCE

RATIO. The sample size of battles from the ACSDB (16 battles) is insufficient to allow observations derived from statistical fits to be treated as definitive or reliable. Also, the eight battles in the bulge area of the theater for each case were components of the other eight theater battles used in that case, thereby creating dependence relationships between sample values. Therefore, derived observations are heavily speculative and should only be treated as suggestive. Additional analysis with other, larger samples is needed to check, supplement, or modify results. The following overall observations are based on the results of the exponential-form trend line fits between FER and FR derived from the ACSDB:

a. Based on coefficient of determination, R^2 , as a measure of goodness of fit, system FER and its associated FR, as derived from the ACSDB, are strongly related by an exponential-form relationship when FER is based on losses in at least (combined) tanks, AT weapons, and artillery systems ($R^2 > .86$). Personnel FER and FR derived from the ACSDB also have a strong exponential-form relationship if FER is based on only combat losses (KIA, WIA, and CMIA) ($R^2 = .90$).

b. Based on percentage error in fitted FER, system FER based on damaged, destroyed, and abandoned tanks, AT/Ms, and artillery has the best exponential fit to FR (15 percent average error). With this criterion, all system FER cases based on a damage criterion of damaged, destroyed, or abandoned, and using losses in at least (combined) tanks, AT weapons, and artillery systems had almost equally good fits (15 - 17 percent average error). Personnel FER based on combat casualties also had a good fit (17 percent average error).

c. The results suggest that a measure of battle outcome results (FER) from the ACSDB may be closely approximated by a simple exponential-form function of initial battle conditions (force ratio). However, the defining parameters of that function are almost certainly scenario-dependent.

d. The results suggest that an exponential-form relationship between FER and FR, when FER is based on a damage criterion of damaged, destroyed, or abandoned systems, including at least tanks, AT weapons, and artillery, may characterize actual combat sufficiently to be useful as a validation criterion for simulated combat in theater models.

CHAPTER 5

RELATIONSHIP OF FER AND FORCE RATIO TO FEBA MOVEMENT

5-1. INTRODUCTION. This chapter presents the results from using the ACSDB to analyze the relationship between FER and FEBA movement, and between force ratio and FEBA movement, using the methodology described in Chapter 2. The FERs and force ratios derived from the ACSDB and the cases studied are the same as those used in Chapter 4. Cases are based on the case attributes listed in Table 2-1. Instead of directly relating FER to FEBA movement, each FER value was first transformed into a measure, denoted as the advantage factor, which is well-known as a major parameter influencing the course of battle for Lanchester-like attrition. This chapter shows:

a. Trend line relationships between the ACSDB-derived advantage factor and FEBA movement. The advantage factor is plotted against FEBA movement for 16 ACSDB battles in each case, and a linear-form trend line is statistically fitted to the plotted points. Quantitative measures of goodness of fit are then used to assess the nature and strength of relationships between advantage factor and movement and to develop general observations.

b. Trend line relationships between ACSDB-derived force ratios and FEBA movement. Both linear and logarithmic trend lines are fitted to plots of force ratio versus FEBA movement, and the resulting relationships are analyzed in a manner similar to that used with the advantage factor.

Results for all cases studied are included in tables shown, but only a selection of graphical results is presented here. Graphical results for most cases analyzed are displayed in Appendices F and G.

5-2. ADVANTAGE FACTOR AND FER. The advantage factor (ADV) and its relationship to FER are described in Chapter 2 of this paper. The definition of ADV used in this chapter is:

$$\begin{aligned} \text{ADV favoring Germans} &= -.5[\text{Ln}(\text{FER favoring US/UK})] && \text{Eq 5-1} \\ &= .5[\text{Ln}(\text{FER favoring Germans})] \end{aligned}$$

where

Ln denotes the natural logarithm function.

Equation 5-1 is only an approximation of the basic definition of advantage factor, given in Equation 2-5 of Chapter 2, but it is usually a good approximation. The ADV value based on personnel for large opposing forces is considered to be an excellent measure of combat effectiveness. ADV values examined in this paper are generalized to include ADV values based on weapon systems as well as personnel.

5-3. ASSESSMENT OF LINEAR TREND LINE RELATIONSHIP BETWEEN ADV AND FEBA MOVEMENT.

A linear regression trend line was fitted to each of 10 scatter plot cases relating ADV to FEBA movement, as derived from the ACSDB. Because of the relationship between ADV and FER expressed in Equation 5-1, each linear regression trend line on a scatter plot of ADV versus FEBA movement is equivalent to a logarithmic trend line fitted to a scatter plot of FER versus FEBA movement. Each scatter plot (of ADV versus FEBA movement) corresponds to one of the FER cases defined in Table 2-1 of Chapter 2 and is characterized by the system mix and damage criterion used to compute the ADV values in the plot. Each scatter plot consists of a set of 16 ADV values plotted against 16 FEBA movement values. Each plotted point is an ordered pair (ADV, KM) where the ADV value in favor of the German side is computed, using Equation 5-1, from the ACSDB, for a 4-day period during the campaign, and KM denotes the historical FEBA movement, in terms of kilometers (km) advanced by the German force during that period. Eight of the (ADV, KM) pairs/points in each plot are computed over the full ARCAS theater, and eight are computed only over the ARCAS bulge. The plotted FEBA movement (KM) is the arithmetic average FEBA displacement along the ARCAS scenario avenues of advance overlaying the theater, or bulge, as appropriate. For each trend line, the value of the coefficient of determination, R^2 , and the average difference between the fitted and historical FEBA movement are used to assess the strength of the fitted relationship between FER and force ratio for each case.

a. Trend Line Results. Tables 5-1 and 5-2 list characteristics of the linear trend line for each of the 10 scatter plot cases.

(1) The cases in Table 5-1 are rank-ordered by descending value of R^2 because the coefficient of determination is a measure of "goodness of fit," with a high value (near 1.00) associated with a strong linear relationship between the plotted ADV and the FEBA movement. In addition to the systems mix and damage criteria used to compute ADV, the following are shown for each case in Table 5-1:

(a) The value of the coefficient of determination, R^2 , associated with the fitted linear trend line.

(b) The equation of the fitted linear trend line. In this equation, ADV denotes the advantage factor value in a (4-day) period, computed in favor of the German side and based on the case definition, while KM denotes the FEBA movement in terms of the (average) number of kilometers which the Germans advanced during the period. Negative KM values denote retrograde German movement.

(c) The ADV value corresponding to no (0 km) FEBA movement on the fitted linear trend line. In the table, this is denoted as the equilibrium point for ADV in fit, because the movements of the opposing forces in that instance are static, equal, and balanced.

Table 5-1. Linear Trend Line Relationship between Historical ADV Value and FEBA Movement

Systems used in ADV	Damage criteria	R²	Equation of fitted line	Equilibrium pt for ADV in fit
Tanks, APCs, AT/Ms, arty	Destroyed or abandoned	.60	KM = 19.06(ADV) +16.02	-0.85
Tanks, AT wpns, arty	Destroyed or abandoned	.60	KM = 20.69(ADV) +12.96	-0.60
Tanks, AT/Ms, arty	Destroyed or abandoned	.59	KM = 21.17(ADV) +13.17	-0.60
Tanks, AT wpns, arty	Damaged, destroyed, or abandoned	.48	KM = 34.37(ADV) +15.90	-0.45
Personnel	Combat casualties	.47	KM = 28.23(ADV) +5.80	-0.20
Tanks, AT/Ms, arty	Damaged, destroyed, or abandoned	.47	KM = 36.11(ADV) +16.58	-0.45
Tanks, APCs, AT/Ms, arty	Damaged, destroyed, or abandoned	.46	KM = 27.31(ADV) +18.86	-0.70
Tanks only	Destroyed or abandoned	.36	KM = 23.34(ADV) +12.70	-0.55
Personnel	Total casualties	.37	KM = 29.33(ADV) +2.54	-0.10
Tanks only	Damaged, destroyed, or abandoned	.28	KM = 35.56(ADV) +16.37	-0.45

(2) Table 5-2 shows the arithmetic average of the errors (in km) in fitted FEBA movements resulting from applying the fitting equation to the 16 historical ADV values plotted for the case. The error is defined as the absolute value of the difference {fitted [KM advanced] - historical [KM advanced]} where KM advanced denotes FEBA movement. The standard deviation of these errors is also shown. The cases in Table 5-2 are in the same case order as used for Table 5-1.

Table 5-2. Average Error in Linear-fitted FEBA Movement Derived from Advantage Factor

Systems used in ADV	Damage criteria	Avg error (km) in fitted FEBA movement	Std dev of error in fitted FEBA movement
Tanks, APCs, AT/Ms, arty	Destroyed or abandoned	5.8	7.0
Tanks, AT wpns, arty	Destroyed or abandoned	6.0	6.8
Tanks, AT/Ms, arty	Destroyed or abandoned	6.0	7.0
Tanks, AT wpns, arty	Damaged , destroyed, or abandoned	6.8	7.7
Personnel	Combat casualties	6.8	7.9
Tanks, AT/Ms, arty	Damaged, destroyed, or abandoned	6.8	8.0
Tanks, APCs, AT/Ms, arty	Damaged, destroyed, or abandoned	6.6	8.3
Tanks only	Destroyed or abandoned	7.5	8.7
Personnel	Total casualties	7.4	8.7
Tanks only	Damaged , destroyed or abandoned	8.6	8.6

b. Case Differences among Linear Fits

(1) Goodness of Fit Based on Coefficient of Determination. Basing strength of the indicated relationship only on the value of R^2 :

(a) The highest ranked relationship in Table 5-1 is for the case computing ADV (and FER) using (including) all major weapon systems with a damage criterion based on only destroyed or abandoned systems. In this case, the value of $R^2 = .60$ indicates a probable relationship between ADV value and FEBA movement using the fitted linear equation.

(b) The system ADV cases using [tanks, AT/Ms, artillery] and [tanks, AT weapons, artillery] and using only destroyed and abandoned systems had R^2 values very close to that of the best fit case.

(c) All ADV values for cases using only destroyed and abandoned systems had better fits to FEBA movement than the corresponding cases based on calculations which also included damaged systems, but differences were not great.

(d) Personnel ADV values using only combat casualties produced a slightly better fit than personnel ADV values using total casualties.

(e) The weakest fits were for cases using only tanks in ADV (and FER) calculations.

(2) Goodness of Fit Based on Percent Error in Fitted FER. An alternative to use of R^2 as an indicator of goodness of fit is to use the absolute error in fitted FEBA movement relative to historical FEBA movement to assess the fit. Ranking by this criterion:

(a) Yields a similar case rank-ordering to that resulting from use of R^2 as a criterion. In this case, the best fit results (5.8 km average error) are again in the system ADV case using all major systems based on only destroyed or abandoned systems. The average historical FEBA movement over the 16 time periods plotted is 9.9 km. The average error in this best fit case is 59 percent of the average historical FEBA movement.

(b) Shows slightly better fits to FEBA movement in cases using only destroyed and abandoned systems than in corresponding cases using damaged, destroyed, and abandoned systems. (Average error was about 1 km less using only destroyed or damaged systems.) Personnel ADV based on combat casualties yielded a slightly better fit than that based on total casualties. These results are similar to those resulting from use of R^2 as a criterion.

Figure 5-1 shows the graphical linear trend line relationships between system ADV and FEBA movement in the two cases using all major systems, which includes the best fit system ADV case (using destroyed and abandoned systems) based on both coefficient of determination and average absolute error in km. The depicted cases differ only in damage criterion. In figures shown herein, DST denotes destroyed or abandoned, and DST & DMGD denotes damaged, destroyed, or abandoned. FEBA movement, shown on the y-axis, is labeled as KM ADVANCE. Positive values for km advance are associated with forward (westward) German movement. Negative values denote retrograde movement. Figure 5-2 shows the graphical linear fit relationship of personnel ADV and FEBA movement in two cases. The depicted cases differ only in damage criterion. In the figure, CBT CAS denotes combat casualties, and TOT CAS denotes total casualties. The figures also show the values of R^2 and the fitting equations for the cases depicted. Graphical results for all cases shown in Table 5-1 are in Appendix F.

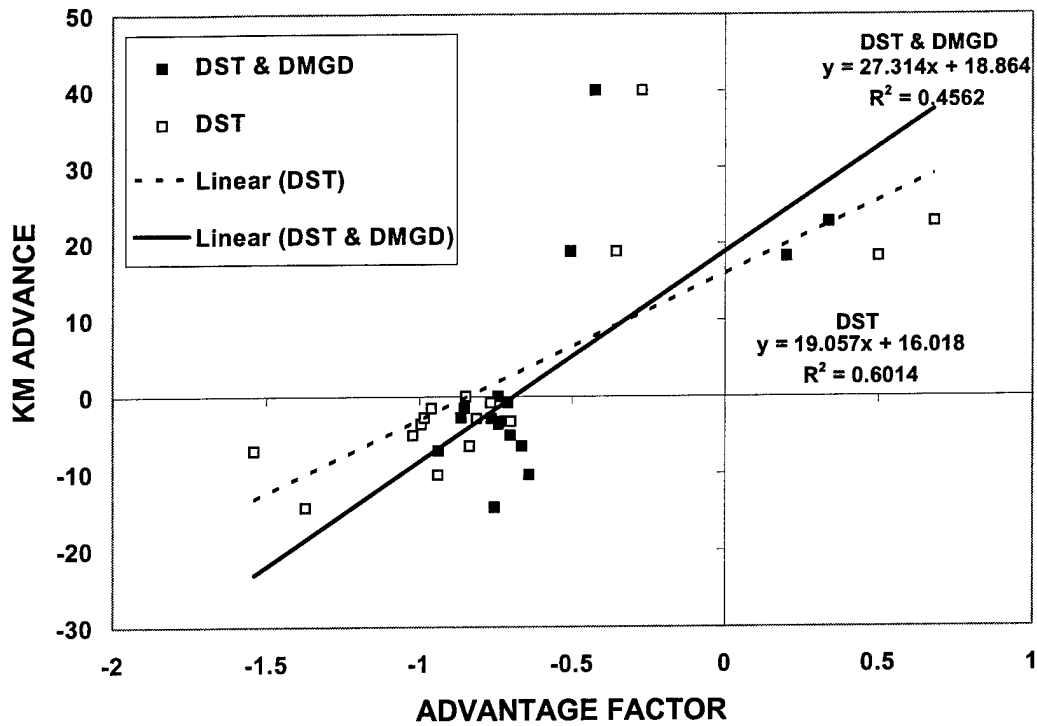


Figure 5-1. Linear Fit of System Advantage Factor vs FEBA Movement (all major systems)

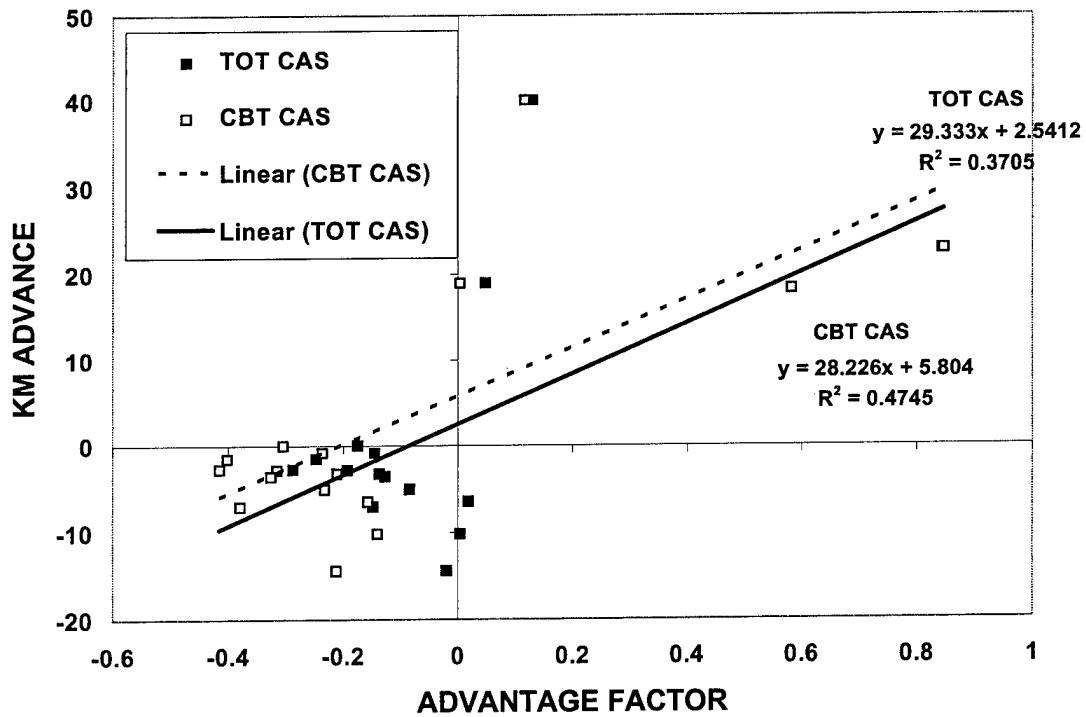


Figure 5-2. Linear Fit of Personnel Advantage Factor vs FEBA Movement

c. Logarithmic Relationship of FER to FEBA Movement. From Equation 5-1, each linear trend line equation for ADV versus KM advance in Table 4-3 is equivalent to a logarithmic trend line equation for FER versus KM advance. For example, using Equation 5-1 to set $ADV = -.5(\ln[FER])$ in the trend line equation on Table 4-3 for the case using all destroyed and abandoned major systems results in the equation:

$$KM = -9.53(\ln[FER]) + 16.02$$

Eq 5-2

The above is the equation of the best fit logarithmic trend line in a scatter plot of FER versus FEBA advance for the case using all destroyed and abandoned major systems. The equivalence is graphically illustrated in Figure 5-3, which is the same data as shown in Figure 5-1, but which is expressed in terms of FER versus FEBA movement, and which has a logarithmic trend line fit, with the trend line equation expressed in the form of Equation 5-2. I choose to relate FER to FEBA movement in terms of the advantage factor approximation because the advantage factor has known theoretical and empirical associations with combat effectiveness.

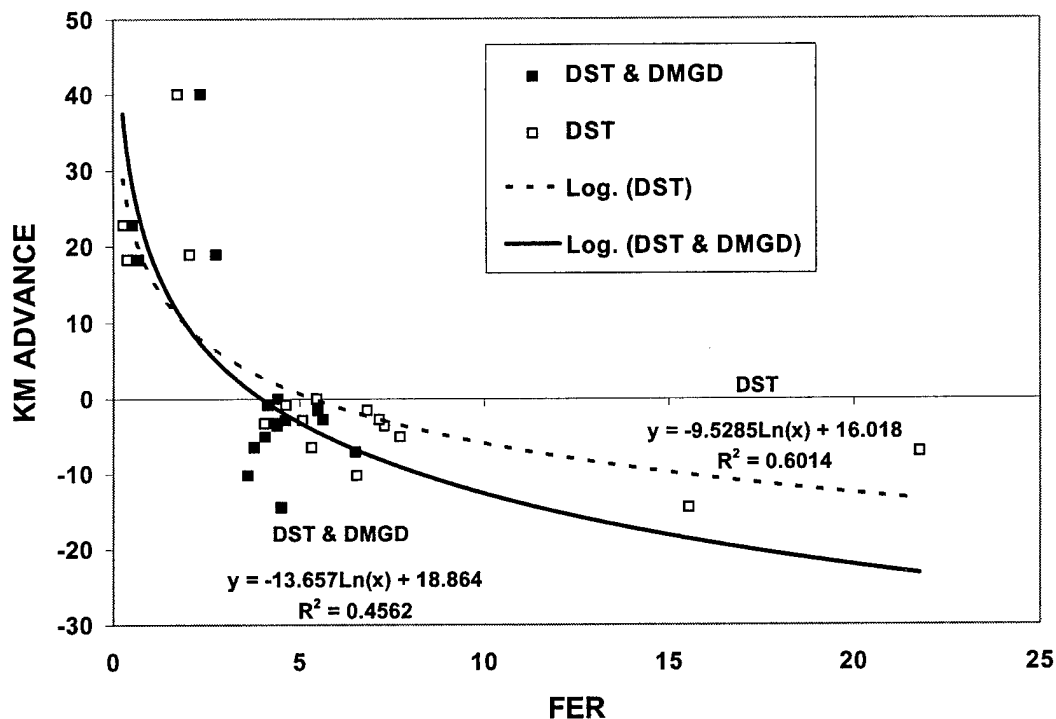


Figure 5-3. Logarithmic Fit of FER vs FEBA Movement (all major systems)

d. Fitted versus Historical FEBA Movement for Linear Fits. Figures 5-4 and 5-5 show graphs of the fitted FEBA movement relative to the historical FEBA movement during the course of the campaign. For each case shown in Table 5-1, fitted FEBA movement (km advance) for each 4-day period in the campaign is determined by the linear fitting equation, using the historical ADV values for that period in that case. The absolute differences between fitted and historical km advance are used to calculate the average percent error in fitted FEBA movement shown in Table 5-2 for each case. Figure 5-4 shows the linearly fitted FEBA

movement plotted at 4-day intervals against the historical FEBA movement, for the best fit case (ADV using all major systems with a damage criterion of destroyed or abandoned) based on both coefficient of determination and average percent error in fitted movement. Each fitted FER value for a period plotted in Figure 5-4 is a y-axis coordinate (km advance) of the point on the dashed (DST) line of Figure 5-1 which has an x-axis (ADV) value equal to the historical advantage factor for that period. Figure 5-5 shows the linearly fitted FEBA movement plotted at 4-day intervals against the historical FEBA movement, for the case with personnel ADV and a damage criterion of combat casualties. The following observations can be made regarding these illustrated cases:

(1) In both cases, fitted movement in the theater is better (closer to historical movement) than fitted movement in the bulge, and the largest deviation from historical results is in the bulge during the period ending at D+8 (the approximate high water mark of the German attack). Also, the magnitudes of both the fitted and historical bulge FEBA movements are larger than corresponding theater FEBA movements in all periods during the first half of the scenario. However, during the last half of the scenario, fitted bulge movement is usually less than fitted theater movement, while historical bulge movement consistently exceeds theater movement during that part of the campaign.

(2) Both cases are very similar during the first half of the scenario, but the ADV value using destroyed and abandoned major systems produces a distinctly better fit to history in the second half than does the personnel ADV using combat casualties.

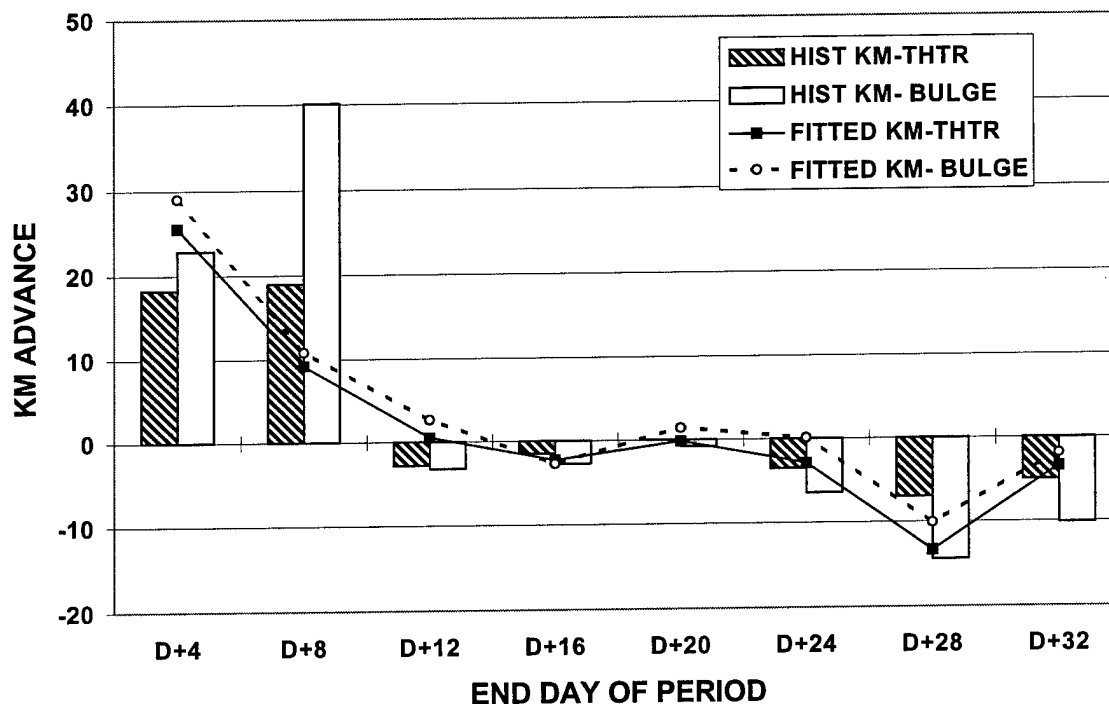


Figure 5-4. Historical FEBA Movement vs Linearly Fitted FEBA Movement
(all destroyed and abandoned major systems)

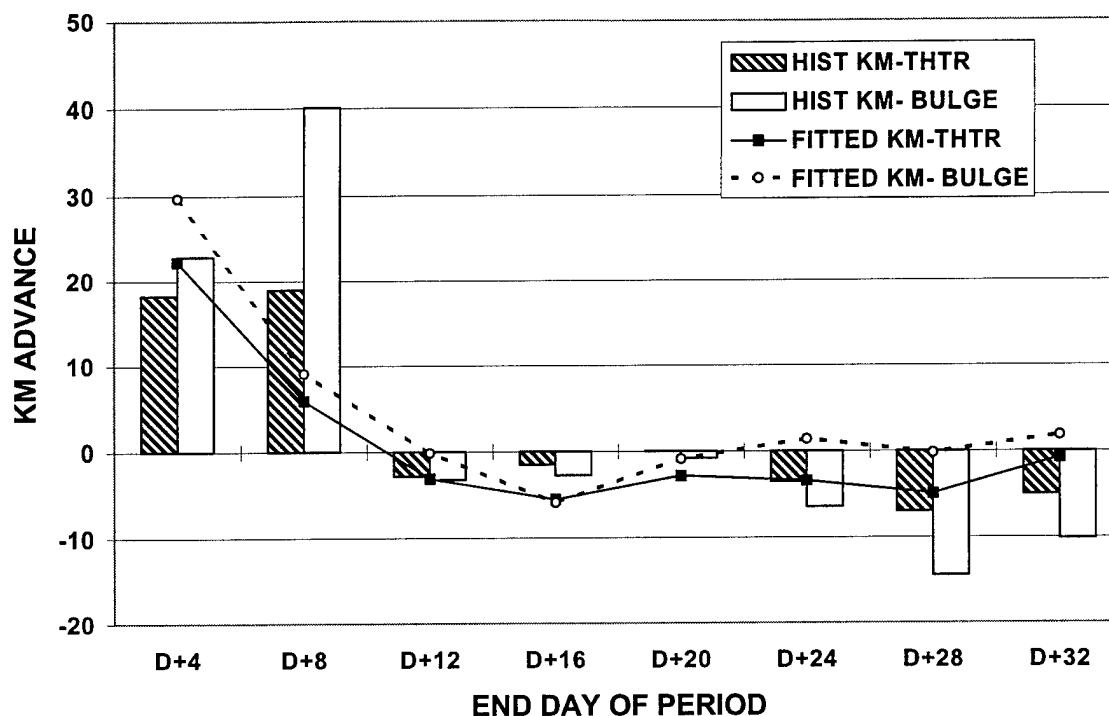


Figure 5-5. Historical FEBA Movement vs Linearly Fitted FEBA Movement (personnel ADV and combat casualties)

e. ADV Equilibrium Point and FEBA Movement. In Table 5-1, the tabulated ADV equilibrium point for a case denotes the ADV value corresponding to no (0 km) FEBA movement on the fitted linear trend line. It is the ADV value for which fitted FEBA movement is zero; i.e., the opposing forces are in a “draw” with respect to movement. However, an ADV value of 0 is associated with a FER equal to 1.00, which is associated with both opposing forces losing systems (or personnel) at equal rates; i.e., an ADV equal to 1.00 is associated with a “draw” with respect to attrition. Negative ADV values in favor of the attacker are, by Equation 5-1, equivalent to FER values in favor of the attacker which are less than 1.00, a situation in which the attacker is losing assets at a greater rate than the defender. Similarly, positive ADV values in favor of the attacker are associated with FER values in favor of the attacker which are greater than 1.00, where the attacker is taking losses at a lesser rate than the defender. Since each linear trend line of ADV versus FEBA movement is equivalent, via Equation 5-1, to a logarithmic trend line of FER versus FEBA movement, each tabulated equilibrium point for ADV in Table 5-1 is equivalent to an equilibrium point for FER on the logarithmic trend line for FER versus FEBA movement. Table 5-3 displays these FER equilibrium points for each case in Table 5-1. Each ADV in Table 5-3 is computed in favor of the Germans, while each FER is computed in favor of the US/UK.

Table 5-3. FER Equilibrium Points Associated with Each ADV Equilibrium Point

Systems used in ADV	Damage criteria	Equilibrium pt for ADV in linear fit of ADV vs KM	Equilibrium pt for FER in equivalent logarithmic fit of FER vs KM
Tanks, APCs, AT/Ms, arty	Destroyed & abandoned	-0.85	5.5
Tanks, AT wpns, arty	Destroyed & abandoned	-0.60	3.5
Tanks, AT/Ms, arty	Destroyed & abandoned	-0.60	3.5
Tanks, AT wpns, arty	Damaged, destroyed, & abandoned	-0.45	2.5
Personnel	Combat casualties	-0.20	1.5
Tanks, AT/Ms, arty	Damaged, destroyed, & abandoned	-0.45	2.5
Tanks, APCs, AT/Ms, arty	Damaged, destroyed, & abandoned	-0.70	4.0
Tanks only	Destroyed & abandoned	-0.55	3.0
Personnel	Total casualties	-0.10	1.2
Tanks only	Damaged, destroyed, & abandoned	-0.45	2.5

(1) Relationship Between ADV Value and Combat Win/Loss. It has been empirically demonstrated that, for large battles, there is a rather steep rise in the probability of a defender win as the ADV value computed in favor of the defender rises from negative to positive values (or, equivalently, as the ADV computed in favor of the attacker falls from positive to negative values). This is intuitively plausible because, in such a case, the definition of ADV implies that a positive ADV in favor of the attacker is associated with the attacker losing systems (or personnel) at a lesser (slower) rate than the defender, while a negative ADV is associated with the attacker's assets wasting away at a faster rate than the defender's. This empirical relationship between ADV and combat effectiveness has, however, been demonstrated only for large battles with ADV defined in terms of personnel casualties.

(2) Relationship Between ADV Value, FER, and Movement. There is no theoretical or empirical foundation showing the ADV, or FER, to be a significant or meaningful predictor of FEBA movement. Force movement is a result of many small battles, with FEBA displacement being a function of combat posture, terrain, relative combat capability of opposing forces, and the amount and nature of logistical, intelligence, and morale factors. Limited logistical support will often constrain the size of an attacker advance more than unfavorable attacker attrition rates. The ADV value, essentially equivalent to the FER, directly reflects only the relative attrition of opposing forces and thus is only a partial determinant of movement. A force may well advance at a substantial rate while sustaining losses at a significantly higher rate than its opponent

because it is in an attack posture with favorable terrain conditions. Conversely, a force losing assets at a far lesser rate than its opponent may well not move at all because it is in a well-fortified defense posture. In the Ardennes scenario, the German force was generally attacking during the first half of the scenario, while the US/UK force was counterattacking during the last half of the scenario. The following observations can be drawn from the equilibrium point results:

(a) ADV equilibrium points for all cases are less than 0.00, with personnel ADV equilibrium points being closest to 0.00 and system ADV equilibrium points for all major systems having the largest negative magnitude. Since ADV is computed in favor of the Germans, this implies that the historical FEBA movement as a function of ADV is biased in favor of the Germans. Specifically, the Germans moved forward further and faster than the US/UK under comparably favorable conditions of relative (attacker/opponent) combat potential, as measured by ADV. Comparable unfavorable conditions induced slower and smaller retrograde movement on the Germans than on the US/UK. The bias is especially large for ADV based on all major systems.

(b) FER equilibrium points for all cases are larger than 1.00, with personnel ADV equilibrium points being closest to 1.00, and system FER equilibrium points for all major systems being largest. FEBA movement as a function of FER is biased in favor of the Germans. The Germans, unlike the US/UK, could advance even when losing assets at a considerably higher rate than their opponents. However, the differences in FER equilibrium points indicates that some asset types were more critical than others in their impact on the FEBA advance. Specifically, the ordering of asset types, from most critical to least critical is:

1. Personnel. Unfavorable FER values of as little as 1.2 to 1.5 were associated with retrograde movement.

2. Tanks. Unfavorable FER values of as little as 2.5 to 3.0 were associated with retrograde movement.

3. Nontank AT weapons and artillery. The crossover point from forward to retrograde movement was associated with FER of at least 2.5 to 3.5.

4. APCs. The crossover point from forward to retrograde movement was associated with an associated FER of at least 4 to 5.5.

5-4. ASSESSMENT OF TREND LINE RELATIONSHIP BETWEEN FORCE RATIO AND FEBA MOVEMENT. Having assessed the empirical relationship between FER and FEBA movement through use of the ADV, completeness requires that we also assess the empirical relationship between force ratio and FEBA movement. Both a linear trend line and a logarithmic trend line are fitted to each of the five scatter plot cases of force ratio (FR) versus FEBA movement corresponding to the five force ratio cases defined in Table 2-2. Since damage criteria are not applicable to force ratio, there is only one personnel force ratio case, and there are four system force ratio cases corresponding to the same four system mixes used in the FER cases. Each plotted point is an ordered pair (FR, KM) where the force ratio (FR) in favor of the German side is computed as the ratio of the onhand German assets to the onhand US/UK assets

at the start of the period and KM denotes the FEBA movement in the period. Eight of the (FR, KM) pairs/points in each plot are computed over the full ARCAS theater, and eight are computed only over the ARCAS bulge

a. Trend Line results. Table 5-4 lists characteristics of the best fit linear trend line for each of the 10 scatter plot cases in order of decreasing value of R^2 . Table 5-5 lists characteristics of the best fit logarithmic trend line for the same cases in order of decreasing value of R^2 . The following are shown for each case:

- (1) The system mix associated with the case (if applicable).
- (2) The value of the coefficient of determination, R^2 , associated with the fitted linear trend line.
- (3) The equation of the fitted linear trend line.
- (4) The average error (in km) and the standard deviation of the errors in each fitted FEBA movement from the fitting equation applied to each of the 16 historical force ratios plotted for the case. The fitting error is computed as the absolute value of the difference {fitted [KM advanced] - historical [KM advanced]}, where KM advanced denotes the average FEBA movement.
- (5) The force ratio value corresponding to no (0 km) FEBA movement on the fitted trend line, denoted as the equilibrium point for the fit.

Table 5-4. Linear Trend Line Relationship between Historical Force Ratio and FEBA Movement

Systems used in force ratio	R^2	Equation of fitted line	Avg error (km) in fitted FEBA advance	Std dev of error in fitted FEBA advance	Equilibrium pt for FR in fit
Tanks, AT/Ms, arty	.54	KM = 22.4(FR)- 13.10	6.1	7.6	0.60
Tanks, APCs, AT/Ms, arty	.54	KM = 27.91(FR)- 12.87	6.2	7.6	.0.50
Tanks, AT wpns, arty	.53	KM = 19.73(FR) - 11.59	6.1	7.7	0.60
Personnel	.47	KM = 16.61(FR) - 15.51	6.5	8.3	0.90
Tanks only	.40	KM = 25.38(FR) - 5.70	7.1	5.4	0.20

Table 5-5. Logarithmic Trend Line Relationship between Historical Force Ratio and FEBA Movement

Systems used in force ratio	R ²	Equation of fitted line	Avg error (km) in fitted FEBA advance	Std dev of error in fitted FEBA advance	Equilibrium pt for FR in fit
Tanks, AT wpns, arty	.64	KM = 20.38Ln(FR)+12.99	6.0	6.2	0.50
Tanks, AT/Ms, arty	.63	KM = 21.46Ln(FR)+13.57	6.0	6.4	0.55
Tanks, APCs, AT/Ms, arty	.62	KM = 21.28Ln(FR)+18.51	6.3	6.2	.040
Tanks only	.56	KM = 14.87Ln(FR)+24.00	6.6	6.8	0.20
Personnel	.53	KM = 24.98Ln(FR)+2.49	6.6	7.2	0.90

b. Case Differences in Goodness of Fit. Rank-order of cases by goodness of fit was very similar regardless of whether coefficient of determination or average fitting error was used to quantify goodness of fit. The logarithmic trend line is a slightly better fit than the linear trend line in each case. Specific observations include:

(1) The three highest ranked cases in Tables 5-4 and 5-5 have nearly identical values for R² and average fitting error. These are for the cases basing system force ratio on at least the tanks, AT weapons, and artillery. In this case, the range of values of R² is 0.62-.0.64 for the logarithmic fit, and 0.53-.0.54 for the linear fit, indicating a probable relationship between force ratio value and FEBA movement using, at least, the fitted logarithmic equation.

(2) The other two cases (personnel, tanks only) had values of R², and average fitting error, which were close to those of the best fit cases.

(3) Personnel force ratio values produced a slightly better linear fit, and a slightly worse logarithmic fit, than did force ratio values using only tanks.

Figure 5-6 shows the graphical relationship of system force ratio and FEBA movement, along with the fitted logarithmic trend line in the case with a system mix of tanks, AT weapons, and artillery systems. Figure 5-7 shows the logarithmic-fitted FEBA movement plotted at 4-day intervals against the historical FEBA movement for this case. Figures 5-8 and 5-9 show analogous graphical results for personnel force ratio versus FEBA movement. Graphical results for all cases shown in Table 5-5 are in Appendix G.

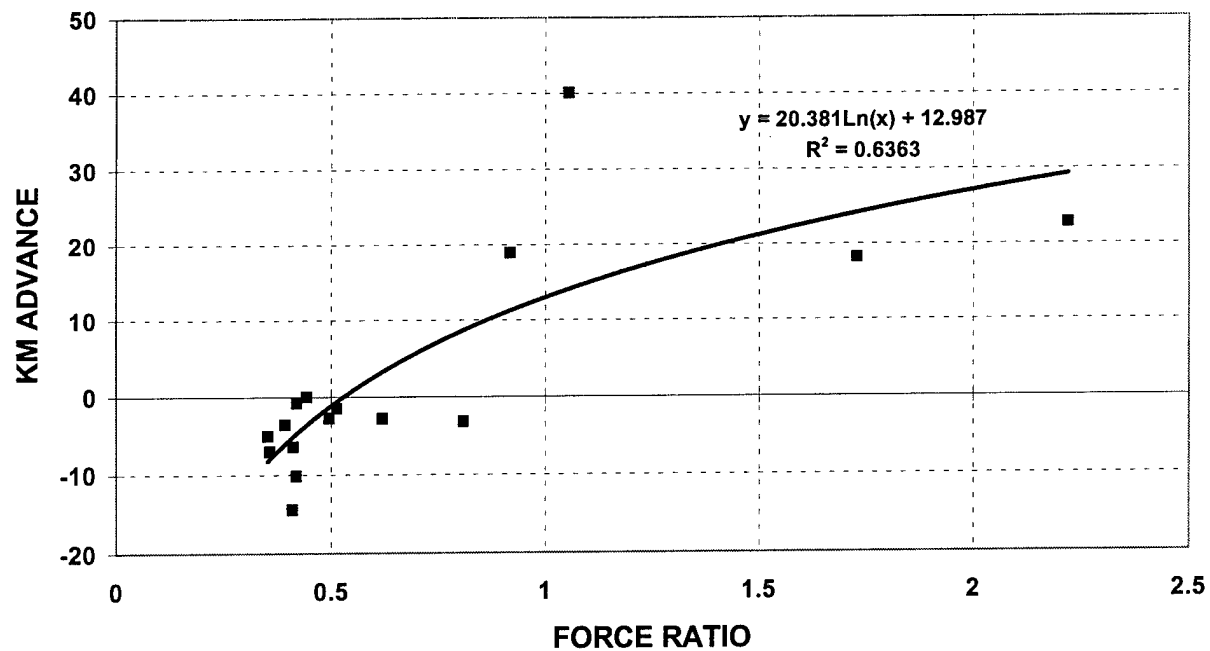


Figure 5-6. Logarithmic Fit of System Force Ratio vs FEBA Movement
(no APCs, mortars)

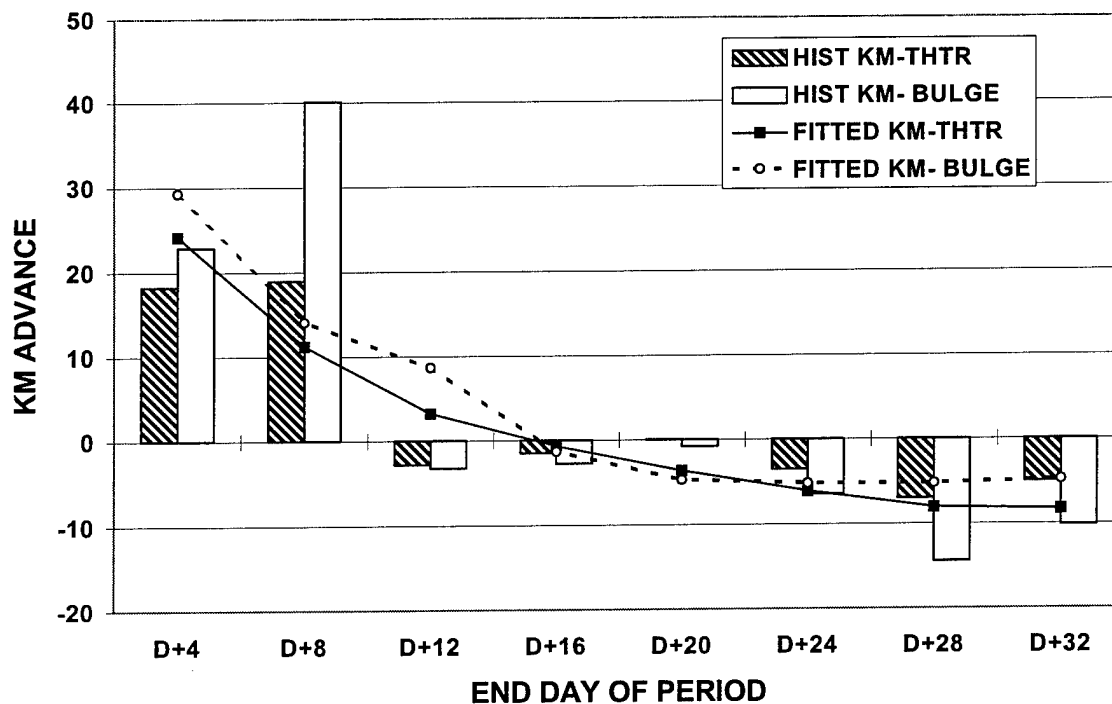


Figure 5-7. Historical FEBA Movement vs Logarithmic Fitted FEBA Movement
(system FR with no APCs, mortars)

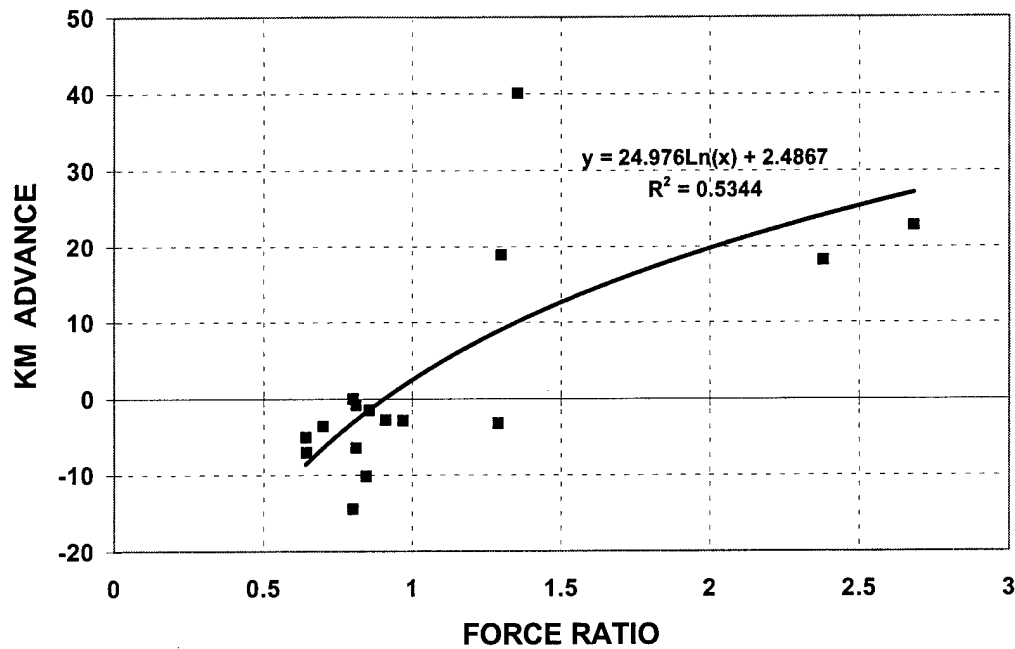


Figure 5-8. Logarithmic Fit of Personnel Force Ratio vs FEBA Movement

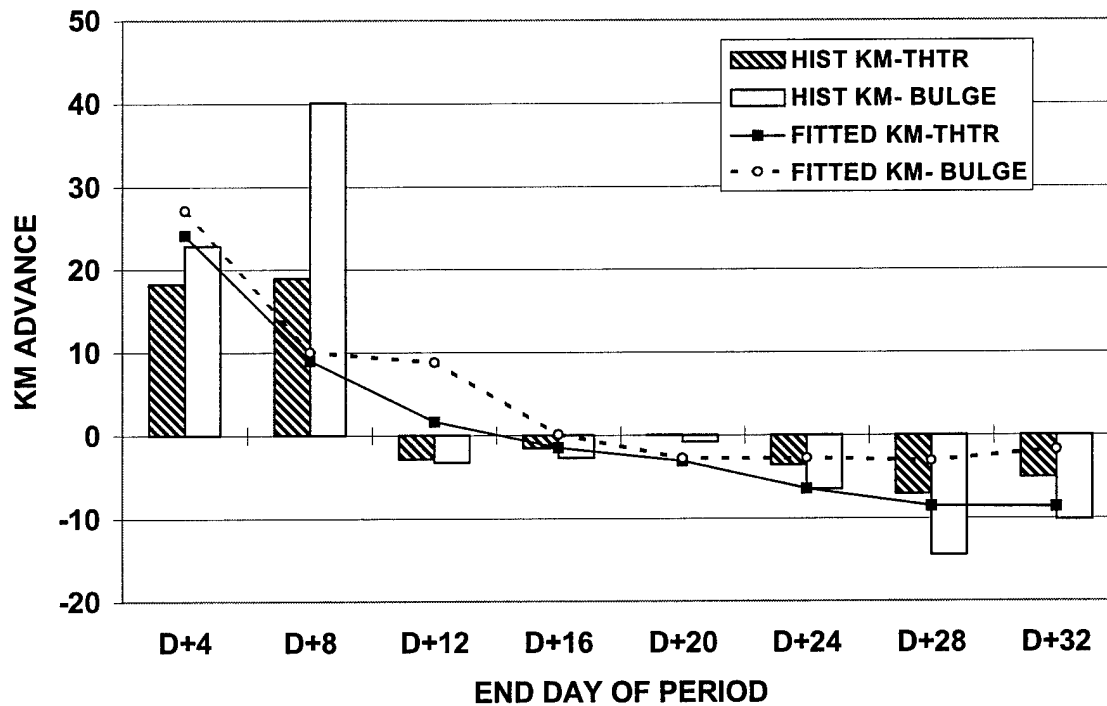


Figure 5-9. Historical FEBA Movement vs Logarithmic Fitted FEBA Movement (personnel force ratio)

CHAPTER 6

DEVELOPMENT OF STANDARDIZATION GUIDELINES FOR A BASIC FER

6-1. PURPOSE. This chapter develops guidelines and suggestions for a standardized computational procedure for a basic FER, based on the survey of CAA FER use presented in Chapter 3 and the ACSDB FER analytic results developed in Chapter 5. Both qualitative and quantitative considerations were used in evaluating candidate FER computational attributes. These recommendations are intended to define a sound, analytically-reasoned, suitable frame of reference for a single basic (system or personnel) FER measure which can be generated, used, shared, and disseminated without ambiguity. They are not intended to restrict model users to a single FER measure in analysis. A model user should always generate a basic FER as a "common denominator" relative to other users, but he/she can freely use whatever additional FER measures are deemed appropriate, as long as those measures are unequivocally defined.

6-2. LIMITATIONS/CAVEATS

a. The FER recommendations presented herein are configured to the needs and customs of model users at CAA and may not be necessary, or appropriate, for another user group/community.

b. The recommendations must be treated as suggestive rather than definitive. The quantitative statistical analyses performed in this study are based on too small a sample to characterize any single recommended basic FER as an unequivocally "best" measure.

6-3. STANDARDIZATION CRITERIA FOR USE/COMPUTATION OF FER. As noted in Chapter 2, the basic definition of FER is open-ended relative to the asset types and loss criteria used in computing FER. One objective of this analysis is to develop guidelines and recommendations for standardizing FER use at CAA. A survey of users, described in Chapter 3, showed that the computational definition of FER differs among model users at CAA. Based on the author's experience, the following criteria are deemed desirable for standardization of the computational definition of FER at CAA:

a. **Feasibility.** The FER computation should be feasible for all users; i.e., each user should be able to compute it from the output of his/her model. The standard definition should only specify systems which can be included in the computation; i.e., those which can be extracted or determined from model output.

b. **High Degree of Consistency Over Models.** Each user should, insofar as possible, define, interpret, and apply the components of FER in the same way. Mathematical equivalency of those components is unlikely between models, because algorithms differ from model to model, even though the processes and products of those algorithms may have the same descriptive English language definitions. For example, destroyed tanks may be a product from two different models, but different tank attrition algorithms may produce different tank losses under the same scenarios. It is often impossible to achieve true scenario consistency between

different models because they usually use different types of inputs and algorithmic processes. Therefore, we should realistically strive to approach, rather than achieve, consistency in use of FER over different models.

c. Strong Association with Combat Effectiveness. If different types of FER have different degrees of trend relationship with combat effectiveness, then the FER with the strongest relationship to combat effectiveness should be preferred.

Completeness of weapon system types (inclusion of all modeled systems in computations) is not listed above, because the results of the ACSDB analysis indicate that it is not required to define a useful and meaningful FER with strong associations with combat effectiveness.

6-4. EVALUATION OF ATTRIBUTES FOR STANDARDIZED FER. The following aspects and attributes of computational methods for FER were evaluated in light of the results in Chapters 3, 4, and 5 and the above standardization criteria.

a. Commonality of Systems. The three CAA models in the user survey use somewhat different system mixes in their "standard" FER definitions. All use tanks, vehicular tank-killing systems, attack helicopters, and artillery in FER computations. The ACSDB analysis in this paper supports inclusion of all these systems in a standardized system FER because ACSDB system FERs using at least tanks, vehicular weapons, and artillery were very similar in their relationship to FEBA movement and to force ratio. However, other system types are either treated differently by each CAA model user, or are excluded by all surveyed model users. The following considerations impact on their inclusion or exclusion in a standard FER:

(1) APCs. CAA models include APCs in FER only if they have a mounted antiarmor or anti-aircraft weapon on them. The APCs in the ACSDB had no such capabilities. The ACSDB analysis showed negligible differences in association with FEBA movement between FERs based on all major systems and those excluding only APCs from the FER. Exclusion of APCs without antiarmor capability from a standard FER therefore appears reasonable. APCs with an antiarmor capability should be categorized as vehicular AT weapons and included in system FER.

(2) Mortars. Mortars are used in EAGLE and CEM only when they are large enough in diameter to be equated to artillery, but the definition of "large enough" differs among model users. Only the EAGLE user's inclusion of mortars of greater than 120mm has any documented justification, based on the CFE treatment of mortars. The COSAGE user excludes all mortars in FER computations, regardless of size. The ACSDB FER analysis showed only very small differences in FER values between FERs excluding APCs and FERs excluding both APCs and small mortars (relative to diameters of modern systems). Exclusion of small mortars therefore appears reasonable in a standardized FER. Inclusion of large mortars as equivalent to artillery also appears justifiable. The EAGLE treatment of mortars with diameters greater than 120mm as artillery appears to have the best (or only) justification and should be adopted.

(3) Air Defense Weapons. EAGLE and COSAGE typically include air defense weapons in FER, but these are generally excluded by a CEM user. Although air defense systems are included in the ACSDB, they sustained negligible losses during the campaign and are not included in system mixes studied. Exclusion of air defense weapons from a standard FER computation appears reasonable, since air defense weapons are not typically employed against land systems and since, for reasons noted below, tactical aircraft should probably be excluded from a standard FER.

(4) Tactical (fixed wing) Air. EAGLE always includes tactical aircraft in FER, while COSAGE computes one type of FER including tactical aircraft, and another optional type excluding it. A CEM user typically excludes tactical aircraft from FER. If tactical air kills/effects are included in FER computations, the user should, at minimum, explicitly acknowledge this is the description of results, even if those effects are not separable from effects with other causes. Tactical aircraft data in the ACSDB had insufficient resolution to enable associating either aircraft, or aircraft sorties, with attacks on the front-line forces engaged. The inclusion of ground attack aircraft in FER would also justify including air-to-air aircraft in it as well. The raw number of onhand aircraft assets should be apportioned, or numerically weighted, according to the proportion of time that they support the battle for which the FER is computed. However, both in practice and in modeling, it is very difficult to associate specific aircraft asset items ("bumper numbers") with a specific duration of tactical ground support or counterair support to a specific battle area. Often, neither the tactical role, nor the area of tactical application of a single specific physical aircraft, is sufficiently well-defined over a campaign to enable a practical and standard way of including it in FER computations. In actual combat, even when dedicated fixed wing ground support is provided, it is usually provided in the form of dedicated sorties, which may be apportioned over different physical aircraft, or even different types of aircraft. In modeled combat, theater combat simulations do not typically track aircraft activity by "bumper number." Because of the above complicating factors, there appears to be no practical way to include TACAIR weapons in FER computations from theater simulations in a manner consistent with treatment of other (land) systems.

b. System Loss Criteria. EAGLE and COSAGE users at CAA use damaged, destroyed, and abandoned systems in FER computations. A CEM user includes only destroyed systems (including abandoned systems) in FER. A damage criterion of destroyed or abandoned appears more appropriate for a standardized FER computation because:

(1) The ACSDB analysis of Chapter 4 noted that system FER based on only destroyed and abandoned systems had a consistently stronger empirical relationship to FEBA movement than FER based on damaged, destroyed, and abandoned systems.

(2) Inclusion of damaged systems would introduce losses due to causes other than combat, which would be extraneous to the (combat) effects we are hoping to capture with FER.

(3) Inclusion of damaged systems allows double counting because rapid repair can allow one physical system to be represented as more than one damaged system in reports during a period.

- (4) Inclusion of damaged systems diminishes the homogeneity of item losses.

COSAGE currently computes only the sum of damaged and destroyed systems and does not determine the destroyed systems separately. However, destroyed systems in COSAGE could be computed by multiplying the number of damaged and destroyed systems of each type by the conditional probability the target is destroyed given that it is hit and sustains damage or is destroyed. These conditional probabilities are available, since they are applied to extrapolated COSAGE results during CEM execution.

c. Personnel Loss Criteria. A combat casualty damage criterion appears most appropriate for a standardized personnel FER for reasons similar to those used to support a destroyed or abandoned criterion for system FER.

(1) The ACSDB analysis of Chapter 4 indicates that personnel FER based on combat casualties (killed/captured/missing in action (KCMIA) and WIA) shows a stronger empirical relationship to FEBA movement than personnel FER based on total casualties, which includes DNBI.

(2) Inclusion of DNBI casualties would introduce losses due to causes other than combat, which would be extraneous to the (combat) effects we hope to capture with FER.

Inclusion of WIA does allow double counting, but this study did not evaluate personnel FER with a damage criterion of only KCMIA because emphasis was on system FER.

d. System FER vs Personnel FER. Computation of personnel FER is downplayed by users of CAA models, who prefer a system FER because it includes effects of big money and big killer items. However, the concept and use of personnel FER is more consistent with the propositions underlying Lanchester combat than the system FERs used herein. The FERs studied in this report's ACSDB analysis are really partial system FERs because they do not include all weapon systems engaged in the battle. Yet most of these partial FERs have, in this study, empirically demonstrated just as strong a relationship to FEBA movement as has personnel FER using combat casualties. A personnel FER based on combat casualties appears to be as useful as system FER.

e. Time of Onhand Assessment. In the FER computation, EAGLE and COSAGE use onhand assets assessed at the start of the period. A CEM user assesses them at the end of the period. Onhand status assessed at the start of the period is suitable.

f. Treatment of Returns from Maintenance/Hospital. The onhand status taken at the start of the period should include all onhand assets at that time. Returns during the period should not be added to it, since this would allow double counting of physical items.

6-5. RECOMMENDATIONS FOR STANDARDIZED COMPUTATION OF FER. Table 6-1 summarizes suggested guidelines for standardized system FER and personnel FER based on the above assessment criteria. If only one type FER is computed, the personnel FER appears to be preferred because it is more readily computed and more consistent with Lanchester combat rules.

Table 6-1. Recommendations for Computation/Definition of Basic FER

Type FER	Systems included	Systems excluded	Losses included
System FER	Tanks, vehicular AT weapons, attack helicopters, mortars >120mm, artillery	Man-carried wpns, mortars < 121mm, tacair, AD wpns	Destroyed and abandoned weapons
Personnel FER	NA	NA	KIA, CMIA, WIA

CHAPTER 7

OVERALL OBSERVATIONS AND RECOMMENDATIONS

7-1. INTRODUCTION. This chapter summarizes observations from other chapters and develops overall observations based on results from all aspects of the analysis. First, observations are developed from investigation of each of the following relationships in statistical computations using the ACSDB.

- a. FER vs force ratio.
- b. FER vs combat effectiveness (FEBA movement).
- c. Force ratio vs combat effectiveness (FEBA movement).

Finally, recommendations are presented for a standardized computational definition of a basic FER for CAA model users, derived from the results of the user survey, and of the FER analyses presented herein.

7-2. LIMITATIONS ON APPLICABILITY. The statistical investigations for each case described in this paper are limited to only 16 points in a single (ARCAS/Ardennes) scenario. Neither the results of these investigations, nor the observations and suggestions derived from them, have a sufficient quantitative basis for generalization beyond the ACSDB data. Yet the results of these analyses have value because:

a. Unquantified statistical confidence in results does not detract from the fact that some unmeasured degree of uncertainty has been reduced in the quantitative information/relationships presented. This research effort should not be treated as an exercise in statistical inference, but as an inductive exploratory analysis which seeks quantitative evidence from statistical methods and measures. In the spirit of the scientific method, this evidence has been used to formulate hypotheses. Rather than being ends in themselves, the presented observations and recommendations should serve as guides and spurs to additional work which, over time, will accumulate to provide definitive answers with measurable, and high, confidence. If complete information cannot be provided now, some information is preferable to no information.

b. The empirical results of this paper provide a link, albeit limited to one scenario, between actual combat and equations of combat. This paper fills a need because very little empirical research on combat relationships has been done at the level of battle resolution studied in this paper. Partial and limited results are acceptable in, and often characteristic of, a new or pioneering effort.

The developed observations and suggestions should therefore be used as a basis for further testing rather than for implementation. Development and publication of these results is nevertheless valuable and necessary because they are empirically-based, using a uniquely comprehensive historical data base (the ACSDB). Just as a journey begins with a single step, the

foundation of an inductive combat rule must begin with a single sample and a single scenario. The author hopes that other empirical work may be done to support, refine, and/or refute the observations and recommendations developed herein.

7-3. RELATIONSHIPS BETWEEN FER AND ACSDB COMBAT STATISTICS. Table 7-1 summarizes the observations on ACSDB relationships between FER and force ratio and FEBA movement, which are analyzed in Chapters 4 and 5.

Table 7-1. ACSDB Relationships of FER, Force Ratio, and FEBA Movement

Relationship	Usefulness of relationship	Preferred FER damage criteria	Major restrictions/remarks
FER vs force ratio	Exponential relation may be useful for model validation	Dmgd/dst/abnd systems, combat casualties (personnel)	Similar relation for all system FERs except those w/tanks only
FER vs combat effectiveness (FEBA advance)	FER as advantage factor may indicate approximate FEBA move potential	Dst/abnd systems, combat casualties (personnel)	Biased in favor of Germans. Personnel FER has least bias. FER based on all major systems has largest bias
Force ratio vs combat effectiveness (FEBA advance)	Force ratio is at least as useful as FER for indication of FEBA move potential	N/A	Biased in favor of Germans. Personnel FER has least bias. FER based on [tanks only] has largest bias

a. FER vs Force Ratio. Results from the statistical fit analysis of the relationship between FER and force ratio in the ACSDB suggest that:

(1) System FER and its associated force ratio, as derived from, and reflected in, the ACSDB, are strongly related by an exponential-form relationship when FER is based on losses in at least (combined) tanks, AT weapons, and artillery systems. Personnel FER and force ratio derived from the ACSDB also have a strong exponential-form relationship, if FER is based on only combat losses (KIA, WIA, CMIA).

(2) A measure of effectiveness of battle outcome (FER) in the ACSDB data can be closely approximated by a simple exponential-form function of initial battle conditions (force ratio). However, the defining parameters of that function are almost certainly scenario-dependent.

(3) An exponential-form relationship between FER and force ratio, when FER is based on at least damaged, destroyed, and abandoned (combined) tanks, AT weapons, and artillery, may characterize actual combat sufficiently to be useful as a validation criterion for simulated combat in theater models.

b. FER vs Combat Effectiveness (FEBA movement) in ACSDB Cases. Results from the statistical fit analysis of a linear relationship between advantage factor and FEBA movement in the ACSDB cases are equivalent to analysis of a logarithmic relationship between FER and FEBA movement. These results suggest that:

(1) System FER is a rough indicator of approximate FEBA progress of engaged theater forces in the ACSDB data. The strongest empirical relationships, based on both coefficient of determination and average fitting error, are associated with the ACSDB cases computing FER using at least (combined) tanks, AT weapons, and artillery with a damage criterion based on only destroyed and abandoned systems.

(2) Personnel FER based on combat casualties is nearly as good an indicator of FEBA progress as system FER and appears to be a slightly better indicator than personnel FER based on total casualties.

(3) The weakest fits of FER to FEBA movement in the ACSDB data were for cases using only tanks in calculations.

(4) FEBA movement as a function of FER was biased in favor of the Germans in the ACSDB. The Germans, unlike the US/UK, advanced even when losing assets at a considerably higher rate than their opponents. However, some asset types were more critical than others in their impact on FEBA advance. Personnel FER showed the least bias, indicating that personnel assets were most critical.

c. Force Ratio vs Combat Effectiveness (FEBA movement) in ACSDB Cases. Results from the statistical fit analysis of a logarithmic relationship between force ratio and FEBA movement in the ACSDB cases suggest that:

(1) Force ratio is at least as good an indicator of approximate FEBA progress as is FER. All statistical fits to FEBA movement with force ratio resulted in slightly better (larger) coefficients of determination than fits of movement to FER, but differences were often very small.

(2) All system force ratios analyzed showed very similar strength in their relationship to FEBA movement, based on both coefficient of determination and average fitting error. All system force ratios showed slightly better logarithmic fits to FEBA movement than did personnel force ratio.

(3) FEBA movement as a function of force ratio was biased in favor of the Germans in the ACSDB. The Germans, unlike the US/UK, advanced even when possessing an unfavorable force ratio. Since assets were not numerically weighted in computations, this indicates that a single German asset item typically had more importance for (effect on) FEBA movement than a corresponding US/UK asset item. However, there were considerable differences in importance between German and US/UK asset types relative to their effect on FEBA movement. Personnel force ratio showed the least bias, indicating that German personnel assets were only slightly more effective/important than their US/UK counterparts. Force ratios using only tanks had the

largest bias, indicating that German tank assets were probably more effective, item for item, than their US/UK counterparts.

7-4. RECOMMENDATIONS FOR COMPUTATION OF A BASIC FER. Table 7-2 summarizes suggested guidelines for standardized computation of a basic system FER and a basic personnel FER within CAA based on the analytic results for ACSDB system mixes and damage criteria assessed in Chapter 5, evaluated in light of the following assessment criteria:

- a. Feasibility of generation/computation of the measure.
- b. Consistency in meaning of FER components, when used in different models.
- c. Reasonably strong relationship between FER and combat effectiveness (expressed as FEBA movement).

Table 7-2. Recommendations for Computation/Definition of Basic FER

Type FER	Systems included	Systems excluded	Losses included
System FER	Tanks, vehicular AT weapons, attack helicopters, mortars >120mm, artillery	Man-carried wpns, mortars < 121mm, TACAIR, AD wpns	Destroyed and abandoned weapons
Personnel FER	NA	NA	KIA, CMIA, WIA

All onhand assets in the computation can be assessed at the start of the period. If only one type FER is computed, the personnel FER appears to be preferred because it is more readily computed, uses homogeneous items, and, in the form of the advantage factor, has previously demonstrated an empirical relationship to combat effectiveness.

APPENDIX A
CONTRIBUTORS

A-1. TEAM

a. Research Director

Mr. Walter J. Bauman, Tactical Analysis Division

A-2. PRODUCT REVIEW BOARD

Mr. Ronald J. Iekel, Chairman

LTC(P) William F. Crain

Mr. Franklin E. Womack

APPENDIX B

REQUEST FOR ANALYTICAL SUPPORT

PART 1	REQUEST FOR ANALYTICAL SUPPORT			
	1. Performing Directorate/ Division: TA		2. Account Number: 96130	
	3. Type Effort (Enter one): Mode (Contract=C) <input type="checkbox"/> <input checked="" type="checkbox"/> R		4. Tasking (Enter one): <input type="checkbox"/> I <input type="checkbox"/> F - Formal Directive I - Informal V - Verbal	
	S - Study Q - QRA P - Project R - RAA M - MMS			
	5. Title: Ardennes Force Exchange Ratio Research			
	6. Acronym: ARFERR		7. Date Request Received: 03/20/96	
			8. Date Due: 07/30/96	
	9. Requester/Sponsor (i.e., DCSOPS): Director, CAA		10. Sponsor Division (i.e., SSW, N/A) TA	
	11. Impact on Other Studies, QRA, Projects, RAA: Partially concurrent with KURSK III work			
	12. Product Required: MR			
	13. Estimated Resources Required:		a. Estimated PSM: 2.5	
			b. Estimated Funds:	
	c. Models Req'd: None		d. Other:	
	14. Objective(s)/Abstract: This effort uses the Ardennes Campaign Simulation Data Base (ACSDB) to define and quantify relationships between Force Exchange Ratios and Force Ratios in historical battle results during the 1944-45 Ardennes Campaign. The ARFERR analysis seeks quantitative relationships between forces and casualties which will increase the understanding of the dynamics of combat and which may improve the use and interpretation of combat simulations at CAA.			
	15. Study Director/POC:		Last Name: Bauman	
		First: Walter		
		Date: 03/19/96		
		Signature: <i>Walter J Bauman</i>		
		Phone#: 295-5261		
GO TO BLOCK 20 If this is A STUDY. See Tab C of the Study Directors' Guide for preparation of a Formal Study Directive.				
PART 2	16. Background/Statement of Problem*: Conjectures have been made that there is a simple relationship between Force Exchange Ratio and Force Ratio for forces in conflict. The ACSDB offers a large amount of empirical data to investigate this hypothesis.			
	17. Scope of Work*: Construction of PC programs to extract ACSDB data to define Force Exchange Ratios and Force Ratios for units in conflict. This also requires defining and quantifying "forces in contact" and devising methods for defining and allocating the elements/portions of units (or clusters of units) in contact. Graphical tools will be used to assess and portray relationships investigated.			
	18. Issues for Analysis*: How can unit contact be meaningfully defined and used in the ACSDB? What methods for defining Force Exchange Ratio and Force Ratio are most useful for studying relationships between these two factors? What graphical relationships were found between Force Exchange Ratio and Force Ratio in ACSDB results studied?			
	19. Milestones/Plan of Action*: Define Problem and approaches -30 Mar. Develop methodology and generate interim results 30 Mar -30 May. Refine methodology and generate/select final results 30 May -30 June. Document findings by 30 July.			
	20. Division Chief Concurrence:		Date: 03/26/96	
	21. Sponsor (COL/DA Div Chief) Concurrence:		Date:	
22. Sponsor Comments*:				

APPENDIX C

REFERENCES

1. Ardennes Campaign Simulation (ARCAS), Study Report, CAA SR-95-8, US Army Concepts Analysis Agency, December 1995, AD A 307 014
2. Ardennes Campaign Simulation Data Base (ACSDB) User's Guide, Data Memory Systems, Inc., Contract No. MDA903-87-C-0787, 18 December 1989
3. The Ardennes Campaign Simulation Data Base (ACSDB), Phase II Final Report (deliverable 0002AK), Data Memory Systems, Inc., Vol I of II and Vol II of II, Contract No. MDA903-87-C-0787, 7 February 1990
4. Combat History Analysis Study Effort (CHASE) Progress Report for the Period August 1984 - June 1985, US Army Concepts Analysis Agency, CAA-TP-86-2, August 1986, AD-F 860 122
5. Do Battles and Wars Have a Common Relationship Between Casualties and Victory? US Army Concepts Analysis Agency, CAA-TP-87-16, November 1987, AD-A 196 126

APPENDIX D

ACSDB UNITS USED IN COMPUTATION OF FER AND FORCE RATIO

D-1. OVERVIEW. This appendix supplements Chapter 2 of the report. The sample of statistical measures (FER or force ratio) computed for each ACSDB case studied consists of exactly 16 measures. Eight of these are theater measures and eight are bulge measures. The eight theater measures (FERs and force ratios) are computed for each 4-day period in the Ardennes Campaign and are based on assets and losses from all of the ACSDB line units committed to the Ardennes conflict during that time period. Similarly, the eight bulge measures are calculated based on only those committed line units in the ACSDB which comprise the historical "bulge" in the Ardennes Campaign. The units in the ARCAS bulge consist chiefly of the German units of the 5th Panzer Army and the US/UK units opposing them. For each 4-day period in the campaign, this appendix lists the front-line units whose assets and losses were used in calculating the FER, and force ratio, for each period. The units listed for each period comprise the engaged units in the "battle" for which the indicated FER, or force ratio, is computed.

D-2. BASIS FOR DEFINITION OF BATTLES. Battles, and contact with the enemy, are not explicitly defined in the ACSDB. Each ACSDB unit, on each day, consists of multiple reported geographic points, each with a separate location, but the data base has insufficient information to measure well-defined unit contact boundaries. Special processor programs were constructed to assess, on a daily basis, the degree of contact between units in the ACSDB, based on the proximity of component points of opposing units. The units participating in the theater battle, and in the bulge battle, were determined based on these proximity/contact analyses, on the position and coverage of each unit, and on the duration of unit presence during the period. First, the German units comprising each battle were defined. The opposing US/UK units were then assessed and defined. Partial representation of US/UK units was also assessed, to account for the case when a US/UK unit was not only engaging a German unit comprising the defined battle, but was also engaging German forces outside of the defined battle. Partial representation in the battles represented in this report consists of crediting only half of the assets and losses of a unit in the FER, or force ratio, computations. This simple weighting method, and the large size of the battles represented, in terms of both geographic area and number of participating units, reduces the effects of uncertainty in boundary contact conditions.

D-3. UNIT COMPOSITION OF THEATER AND BULGE BATTLES. Tables D-1 and D-2 show, for each 4-day period in the campaign, the German and US/UK units from the ACSDB which were used to compute theater FER and force ratio. Tables D-3 and D-4 show analogous unit composition for each battle used to compute bulge FER and force ratio. The units listed for each battle consist of the committed front-line units in the defined (theater or bulge) battle area, assessed from the ACSDB as described above. Definitions of abbreviated unit name components are found in the glossary. US/UK units are prefixed by (B). Units with partial representation have names enclosed in brackets.

Table D-1. German Units Used to Compute Theater FER and Force Ratio in Each Period

End day of period							
D+4	D+8	D+12	D+16	D+20	D+24	D+28	D+32
1st SSPzD	1st SSPzD	1st SSPzD	1st SSPzD	1st SSPzD	1st SSPzD	1st SSPzD	1st SSPzD
12th SSPzD	9th SSPzD	2d SSPzD	2d SSPzD	2d SSPzD	2d SSPzD	2d SSPzD	2d SSPzD
PzLehrD	12th SSPzD	9th SSPzD	9th SSPzD	9th SSPzD	9th SSPzD	9th SSPzD	9th SSPzD
2d PzD	PzLehrD	12th SSPzD	12th SSPzD	12th SSPzD	12th SSPzD	12th SSPzD	12th SSPzD
116th PzD	2d PzD	PzLehrD	PzLehrD	PzLehrD	PzLehrD	PzLehrD	PzLehrD
12th VGD	116th PzD	2d PzD	2d PzD	2d PzD	2d PzD	2d PzD	2d PzD
18th VGD	12th VGD	9th PzD	9th PzD	9th PzD	9th PzD	9th PzD	9th PzD
26th VGD	18th VGD	116th PzD	116th PzD	116th PzD	116th PzD	116th PzD	116th PzD
62d VGD	26th VGD	12th VGD	12th VGD	9th VGD	9th VGD	9th VGD	9th VGD
212th VGD	62d VGD	18th VGD	18th VGD	12th VGD	12th VGD	12th VGD	12th VGD
276th VGD	212th VGD	26th VGD	26th VGD	18th VGD	18th VGD	18th VGD	18th VGD
277th VGD	276th VGD	62d VGD	62d VGD	26th VGD	26th VGD	26th VGD	26th VGD
352d VGD	277th VGD	79th VGD	79th VGD	62d VGD	62d VGD	62d VGD	62d VGD
560th VGD	340th VGD	212th VGD	212th VGD	79th VGD	79th VGD	79th VGD	79th VGD
3d FJD	352d VGD	276th VGD	276th VGD	167th VGD	167th VGD	167th VGD	167th VGD
5th FJD	560th VGD	277th VGD	277th VGD	212th VGD	212th VGD	212th VGD	212th VGD
	3d PzGD	340th VGD	340th VGD	276th VGD	276th VGD	276th VGD	276th VGD
	3d FJD	352d VGD	352d VGD	277th VGD	277th VGD	277th VGD	277th VGD
	5th FJD	560th VGD	560th VGD	340th VGD	340th VGD	340th VGD	340th VGD
	150th PzBde	3d PzGD	3d PzGD	352d VGD	352d VGD	352d VGD	352d VGD
	FBB	15th PzGD	15th PzGD	560th VGD	560th VGD	560th VGD	560th VGD
		3d FJD	3d FJD	3d PzGD	3d PzGD	3d PzGD	3d PzGD
		5th FJD	5th FJD	15th PzGD	15th PzGD	15th PzGD	15th PzGD
		150th PzBde	150th PzBde	3d FJD	3d FJD	3d FJD	3d FJD
		FBB	FBB	5th FJD	5th FJD	5th FJD	5th FJD
		FGB	FGB	150th PzBde	150th PzBde	150th PzBde	150th PzBde
				FBB	FBB	FBB	FBB
				FGB	FGB	FGB	FGB

Table D-2. US/UK Units Used to Compute Theater FER and Force Ratio in Each Period

End day of period							
D+4	D+8	D+12	D+16	D+20	D+24	D+28	D+32
[7th AD]*	3d AD	2d AD	2d AD	2d AD	2d AD	2d AD	2d AD
9th AD	7th AD	3d AD	3d AD	3d AD	3d AD	3d AD	3d AD
[2d ID]	9th AD	4th AD	4th AD	4th AD	4th AD	4th AD	4th AD
4th ID	10th AD	7th AD	7th AD	6th AD	6th AD	6th AD	6th AD
28th ID	82d AbnD	9th AD	9th AD	7th AD	7th AD	7th AD	7th AD
[99th ID]	101st AbnD	10th AD	10th AD	9th AD	9th AD	9th AD	9th AD
106th ID	1st ID	82d AbnD	82d AbnD	10th AD	10th AD	10th AD	10th AD
	2d ID	101st AbnD	101st AbnD	11th AD	11th AD	11th AD	11th AD
	4th ID	1st ID	1st ID	82d AbnD	17th AbnD	17th AbnD	17th AbnD
	28th ID	2d ID	2d ID	101st AbnD	82d AbnD	82d AbnD	82d AbnD
	30th ID	4th ID	4th ID	1st ID	101st AbnD	101st AbnD	101st AbnD
	[84th ID]	5th ID	5th ID	2d ID	1st ID	1st ID	1st ID
	99th ID	26th ID	26th ID	4th ID	2d ID	2d ID	2d ID
	106th ID	28th ID	28th ID	5th ID	4th ID	4th ID	4th ID
		30th ID	30th ID	26th ID	5th ID	5th ID	5th ID
		75th ID	35th ID	28th ID	26th ID	26th ID	26th ID
		80th ID	75th ID	30th ID	28th ID	28th ID	28th ID
		84th ID	80th ID	35th ID	30th ID	30th ID	30th ID
		99th ID	83d ID	75th ID	35th ID	35th ID	35th ID
		[106th ID]	84th ID	80th ID	75th ID	75th ID	75th ID
		(B)29th ArmBde	[99th ID]	83d ID	80th ID	80th ID	80th ID
			[106th ID]	84th ID	83d ID	83d ID	83d ID
			(B)29th ArmBde	87th ID	84th ID	84th ID	84th ID
				[99th ID]	87th ID	87th ID	87th ID
				[106th ID]	[99th ID]	90th ID	90th ID
				(B)29th ArmBde	[106th ID]	[99th ID]	[99th ID]
					(B)29th ArmBde	[106th ID]	106th ID
					(B)53d ID	(B)29th ArmBde	(B)29th ArmBde
						(B)51st ID	(B)51st ID
						(B)53d ID	(B)53d ID

*Unit names in brackets [] indicate that only 50 percent of unit assets and losses are used in the computations for that period.

Table D-3. German Units Used to Compute Bulge FER and Force Ratio in Each Period

End day of period							
D+4	D+8	D+12	D+16	D+20	D+24	D+28	D+32
1st SSPzD	1st SSPzD	1st SSPzD	1st SSPzD	2d SSPzD	2d SSPzD	2d SSPzD	2d SSPzD
PzLehrD	9th SSPzD	2d SSPzD	2d SSPzD	9th SSPzD	9th SSPzD	9th SSPzD	9th SSPzD
2d PzD	12th SSPzD	9th SSPzD	9th SSPzD	12th SSPzD	12th SSPzD	12th SSPzD	12th SSPzD
116th PzD	PzLehrD	12th SSPzD	12th SSPzD	PzLehrD	PzLehrD	PzLehrD	PzLehrD
12th VGD	2d PzD	PzLehrD	PzLehrD	2d PzD	2d PzD	2d PzD	2d PzD
18th VGD	116th PzD	2d PzD	2d PzD	9th PzD	9th PzD	9th PzD	9th PzD
26th VGD	12th VGD	9th PzD	9th PzD	116th PzD	116th PzD	116th PzD	116th PzD
62d VGD	18th VGD	116th PzD	116th PzD	12th VGD	12th VGD	12th VGD	12th VGD
560th VGD	26th VGD	12th VGD	12th VGD	18th VGD	18th VGD	18th VGD	18th VGD
3d FJD	62d VGD	18th VGD	18th VGD	26th VGD	26th VGD	26th VGD	26th VGD
	560th VGD	26th VGD	26th VGD	62d VGD	62d VGD	62d VGD	62d VGD
	3d FJD	62d VGD	62d VGD	167th VGD	167th VGD	167th VGD	167th VGD
	FBB	560th VGD	560th VGD	340th VGD	340th VGD	340th VGD	340th VGD
		15th PzGD	15th PzGD	560th VGD	560th VGD	560th VGD	560th VGD
		3d FJD	3d FJD	15th PzGD	3d PzGD	3d PzGD	3d PzGD
		FBB	FBB	3d FJD	15th PzGD	15th PzGD	15th PzGD
				FBB	3d FJD	3d FJD	3d FJD
					FBB	FBB	FBB

Table D-4. US/UK Units Used to Compute Bulge FER and Force Ratio in Each Period

End day of period							
D+4	D+8	D+12	D+16	D+20	D+24	D+28	D+32
[7th AD]*	3d AD	2d AD	2d AD	2d AD	2d AD	2d AD	2d AD
28th ID	7th AD	3d AD	3d AD	3d AD	3d AD	3d AD	3d AD
[99th ID]	82d AbnD	7th AD	4th AD	6th AD	6th AD	[4th AD]	6th AD
106th ID	101st AbnD	82d AbnD	7th AD	[7th AD]	9th AD	6th AD	11th AD
	[1st ID]	101st AbnD	9th AD	9th AD	11th AD	17th AbnD	17th AbnD
	[2d ID]	[1st ID]	82d AbnD	11th AD	17th AbnD	82d AbnD	82d AbnD
	28th ID	[2d ID]	101st AbnD	82d AbnD	82d AbnD	101st AbnD	101st AbnD
	30th ID	28th ID	[1st ID]	101st AbnD	101st AbnD	[26th ID]	[1st ID]
	[84th ID]	30th ID	[2d ID]	1st ID	[1st ID]	[30th ID]	[30th ID]
	[99th ID]	84th ID	28th ID	28th ID	30th ID	35th ID	[35th ID]
	106th ID	[99th ID]	[30th ID]	30th ID	[35th ID]	75th ID	75th ID
		(B)29th ArmBde	35th ID	[35th ID]	75th ID	83d ID	83d ID
			75th ID	75th ID	83d ID	84th ID	84th ID
			83d ID	83d ID	84th ID	87th ID	87th ID
			84th ID	84th ID	87th ID	90th ID	90th ID
			(B)29th ArmBde	87th ID	(B)29th ArmBde	[106th ID]	106th ID
				(B)29th ArmBde	(B)53d ID	(B)29th ArmBde	(B)51st ID
						(B)51st ID	

*Unit names in brackets [] indicate that only 50 percent of unit assets and losses are used in the computations for that period.

APPENDIX E

EXPONENTIAL TRENDS BETWEEN FER AND FORCE RATIO

OVERVIEW. This appendix supplements Chapter 4 of the report.

a. Figures E-1 through E-5 show scatter plots with exponential trend lines for FER versus force ratio (FR), as computed from the ACSDB for each of the 10 cases associated with the data tabulated in Table 4-3. Tables E-1 through E-5 tabulate the data plotted in the figures. Figures E-1 and E-2 appear in Chapter 4 as Figures 4-5 and 4-6. Each scatter plot case consists of a set of 16 FERs plotted against 16 force ratios, where each plotted point is an ordered pair (FR, FER) where the FER and FR are computed from the ACSDB, with FER in favor of the US/UK side, and FR in favor of the German side, for a 4-day period during the campaign. Eight of the (FR, FER) pairs/points in each plot are theater FERs, or FRs, computed over the full ARCAS theater, and eight are bulge FERs, or FRs, computed over only the ARCAS bulge. Each scatter plot case corresponds to one of the 10 FER cases described in Table 2-1. A regression trend line, with the exponential-form expressed in Equation 4-1, is fitted to each of the scatter plots. Each scatter plot also shows the fitted trend line, along with the fitted equation, and the value of R^2 , the coefficient of determination, for the trend line. Each figure shows two scatter plots, which differ only in the damage criterion used to compute FER and force ratio. All 32 plotted points in each figure correspond to FERs or force ratios based on the same mix of systems (or personnel). The system mix is identified in the figure label. The two sets of plotted points and trend lines in each figure are labeled according to the associated damage criterion.

b. Figures E-6 through E-15 show the exponential-form fitted FER plotted at 4-day intervals against the historical (ACSDB) FER, for each of the exponential fits in the 10 cases with characteristics tabulated in Table 4-3. Tables E-6 through E-15 tabulate the data plotted in the figures. Each fitted FER is determined by the appropriate fitted equation for that case, using the historical force ratio value for that period and case.

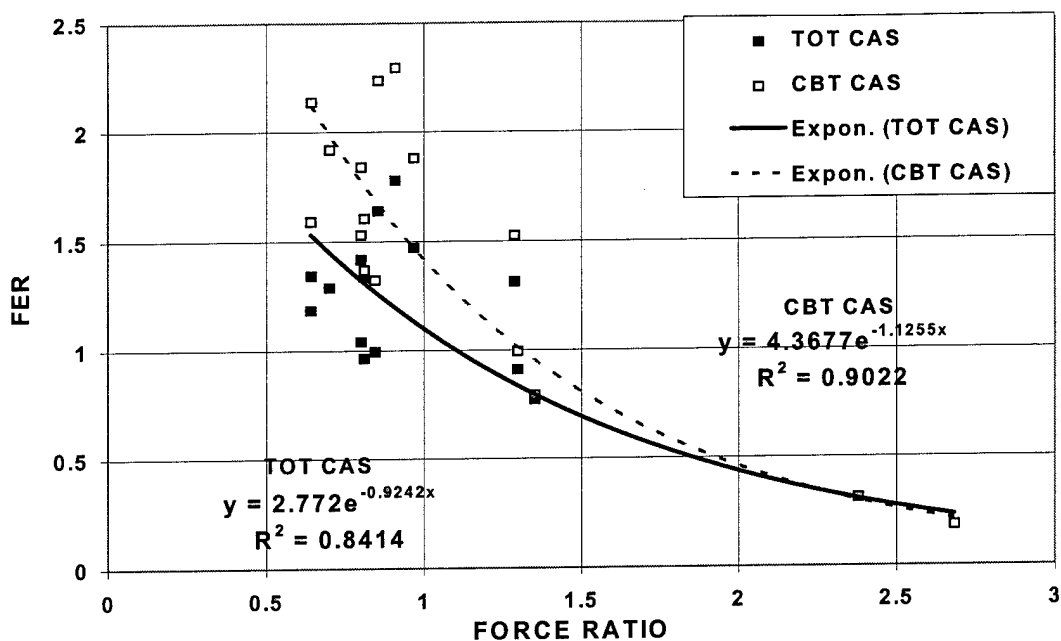


Figure E-1. Exponential Fit of ACSDB Personnel FER vs Force Ratio

Table E-1. Exponential Fit of ACSDB Personnel FER vs Force Ratio

		D+4	D+8	D+12	D+16	D+20	D+24	D+28	D+32
	Theater FR	2.38	1.298	0.968	0.854	0.8	0.7	0.643	0.641
	Bulge FR	2.683	1.353	1.288	0.909	0.81	0.811	0.8	0.845
TOT CAS	Theater FER	0.312	0.909	1.472	1.64	1.418	1.289	1.344	1.184
	Bulge FER	0.185	0.772	1.314	1.778	1.338	0.964	1.04	0.993
CBT CAS	Theater FER	0.312	0.993	1.88	2.235	1.84	1.919	2.137	1.592
	Bulge FER	0.184	0.793	1.525	2.295	1.606	1.37	1.53	1.324

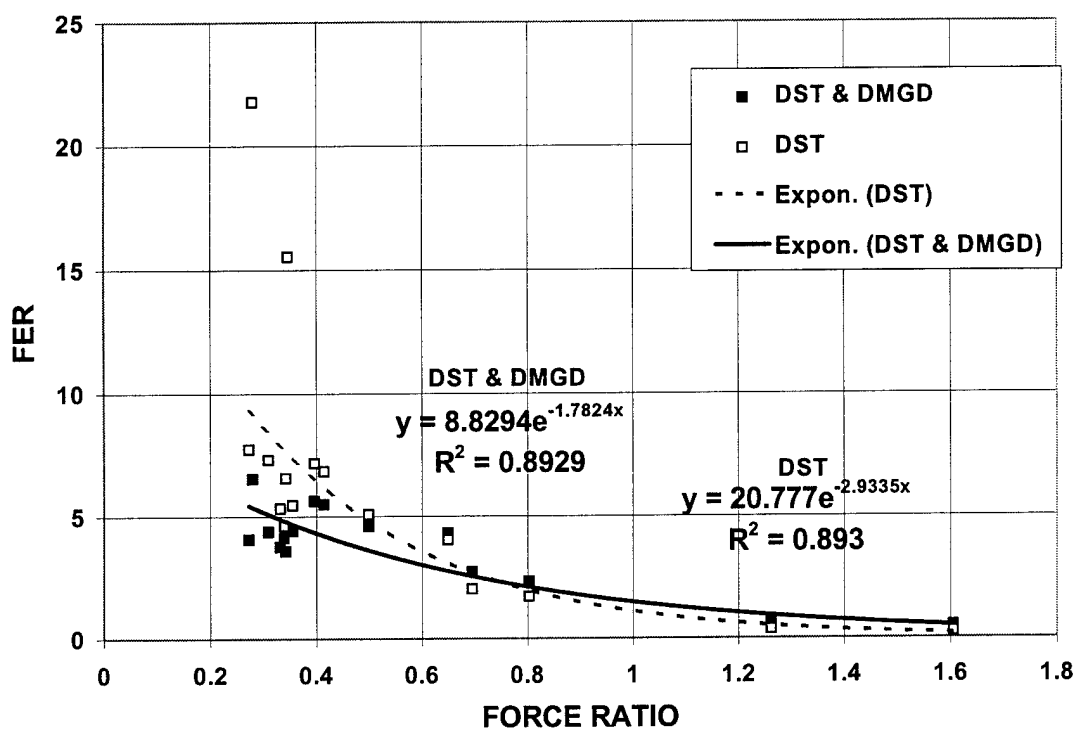


Figure E-2. Exponential Fit of ACSDB System FER vs Force Ratio
(all major systems)

Table E-2. Exponential Fit of ACSDB System FER vs Force Ratio
(all major systems)

		D+4	D+8	D+12	D+16	D+20	D+24	D+28	D+32
	Theater FR	1.262	0.695	0.499	0.414	0.355	0.309	0.279	0.272
	Bulge FR	1.605	0.803	0.649	0.396	0.339	0.332	0.346	0.342
DST & DMGD	Theater FER	0.67	2.748	4.607	5.493	4.417	4.399	6.513	4.075
	Bulge FER	0.509	2.339	4.323	5.631	4.136	3.78	4.512	3.604
DST	Theater FER	0.368	2.04	5.08	6.822	5.459	7.293	21.769	7.715
	Bulge FER	0.256	1.723	4.062	7.151	4.626	5.322	15.541	6.542

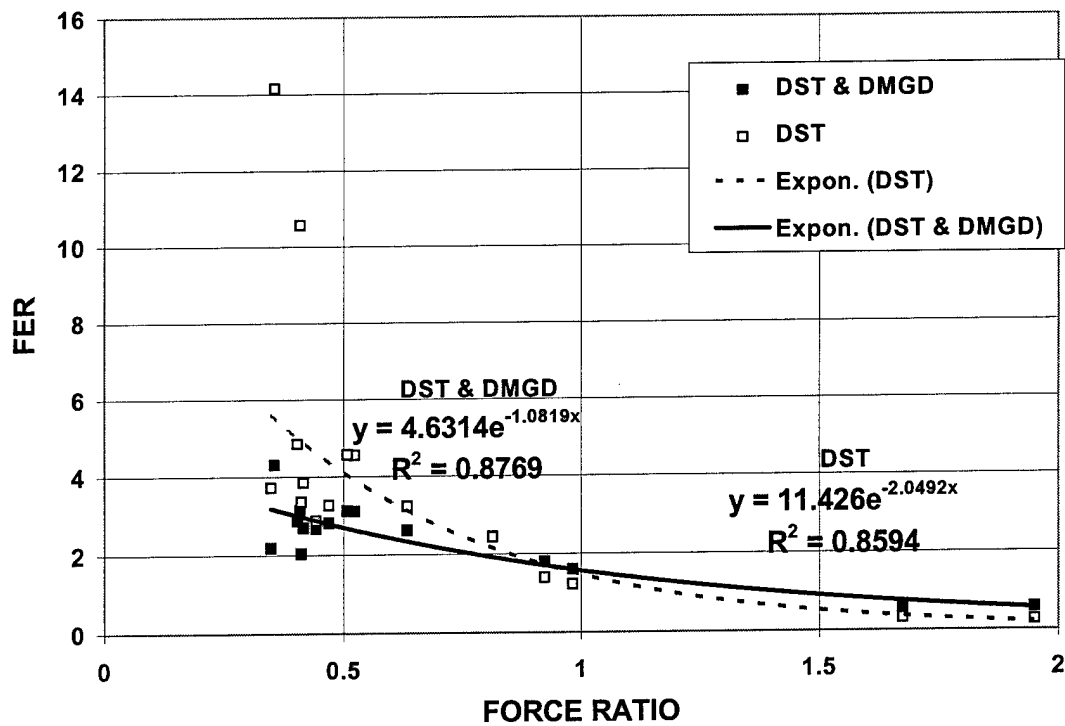


Figure E-3. Exponential Fit of ACSDB System FER vs Force Ratio
(tanks, AT/Ms, arty)

Table E-3. Exponential Fit of ACSDB System FER vs Force Ratio
(tanks, AT/Ms, arty)

		D+4	D+8	D+12	D+16	D+20	D+24	D+28	D+32
	Theater FR	1.675	0.923	0.634	0.523	0.469	0.403	0.355	0.348
	Bulge FR	1.95	0.982	0.813	0.508	0.442	0.416	0.408	0.411
DST & DMGD	Theater FER	0.622	1.81	2.618	3.111	2.813	2.871	4.306	2.184
	Bulge FER	0.581	1.611	2.446	3.125	2.673	2.693	3.135	2.04
DST	Theater FER	0.327	1.403	3.229	4.554	3.261	4.854	14.159	3.719
	Bulge FER	0.265	1.233	2.438	4.573	2.875	3.847	10.576	3.356

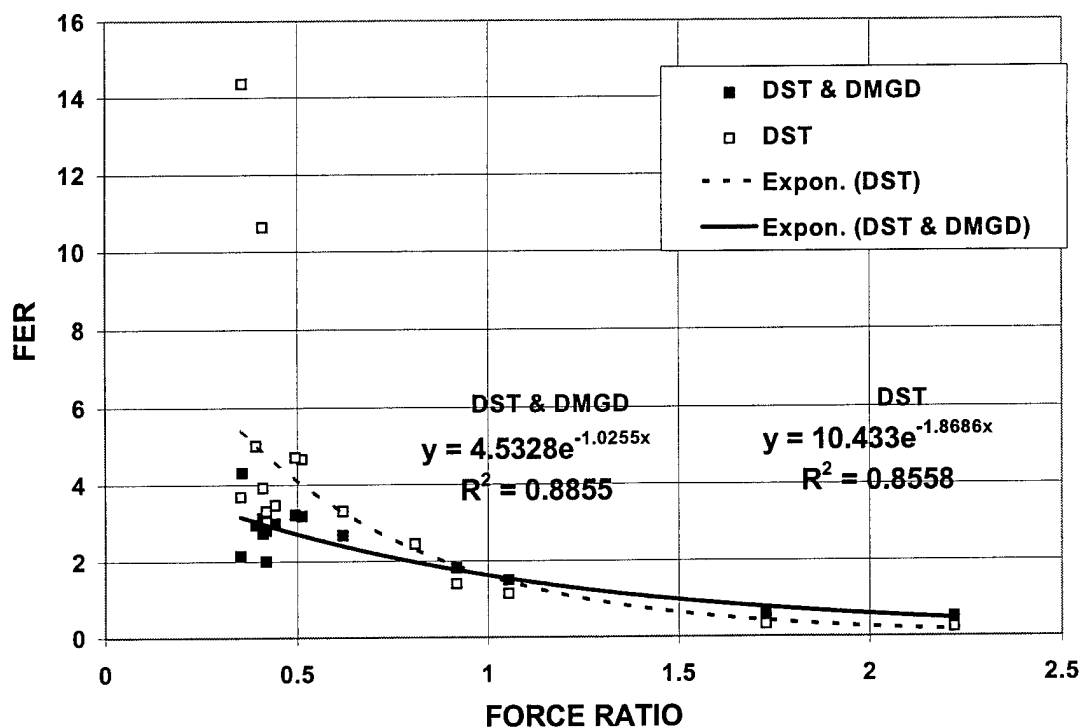


Figure E-4. Exponential Fit of ACSDB System FER vs Force Ratio
(tanks, AT wpns, arty)

Table E-4. Exponential Fit of ACSDB System FER vs Force Ratio
(tanks, AT wpns, arty)

		D+4	D+8	D+12	D+16	D+20	D+24	D+28	D+32
	Theater FR	1.729	0.917	0.619	0.512	0.442	0.392	0.356	0.352
	Bulge FR	2.22	1.054	0.808	0.495	0.419	0.41	0.409	0.418
DST & DMGD	Theater FER	0.604	1.822	2.68	3.181	2.988	2.956	4.311	2.158
	Bulge FER	0.513	1.5	2.461	3.215	2.817	2.737	3.132	2.007
DST	Theater FER	0.318	1.412	3.308	4.669	3.469	5.016	14.359	3.688
	Bulge FER	0.234	1.149	2.453	4.716	3.033	3.918	10.642	3.31

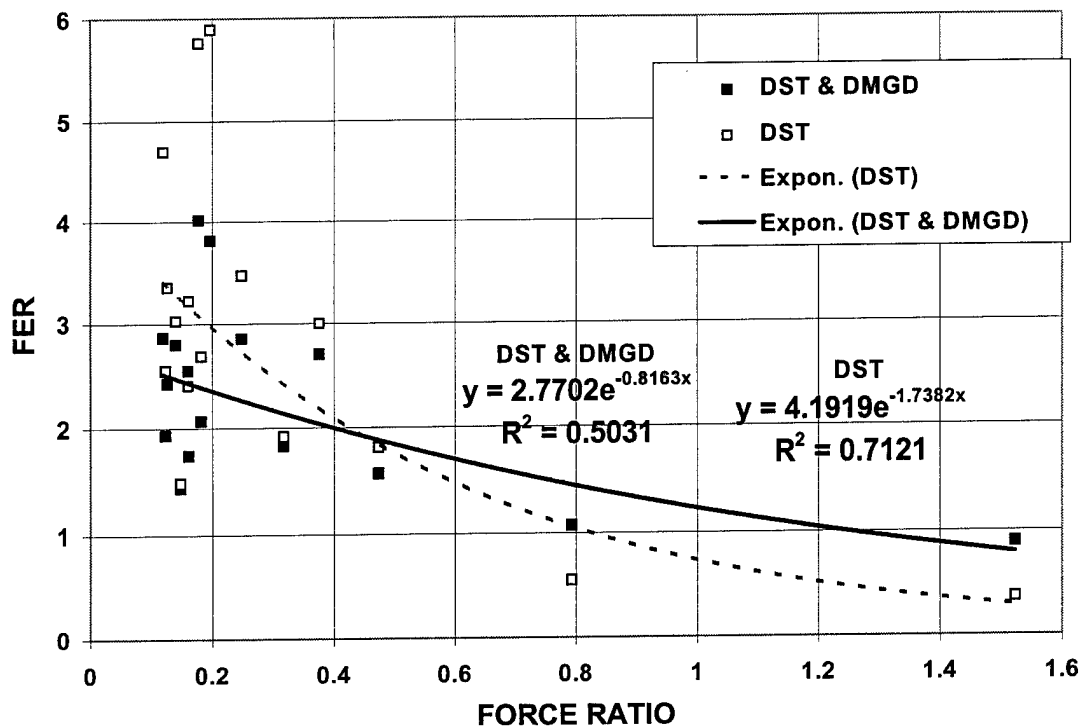


Figure E-5. Exponential Fit of ACSDB System FER vs Force Ratio (using tanks only)

Table E-5. Exponential Fit of ACSDB System FER vs Force Ratio (tanks only)

		D+4	D+8	D+12	D+16	D+20	D+24	D+28	D+32
	Theater FR	0.793	0.317	0.248	0.177	0.14	0.123	0.119	0.126
	Bulge FR	1.523	0.474	0.376	0.196	0.16	0.148	0.161	0.182
DST & DMGD	Theater FER	1.07	1.833	2.853	4.017	2.799	1.935	2.867	2.428
	Bulge FER	0.899	1.569	2.704	3.814	2.55	1.445	1.744	2.068
DST	Theater FER	0.557	1.919	3.469	5.764	3.029	2.549	4.702	3.356
	Bulge FER	0.374	1.816	2.999	5.896	2.406	1.485	3.225	2.685

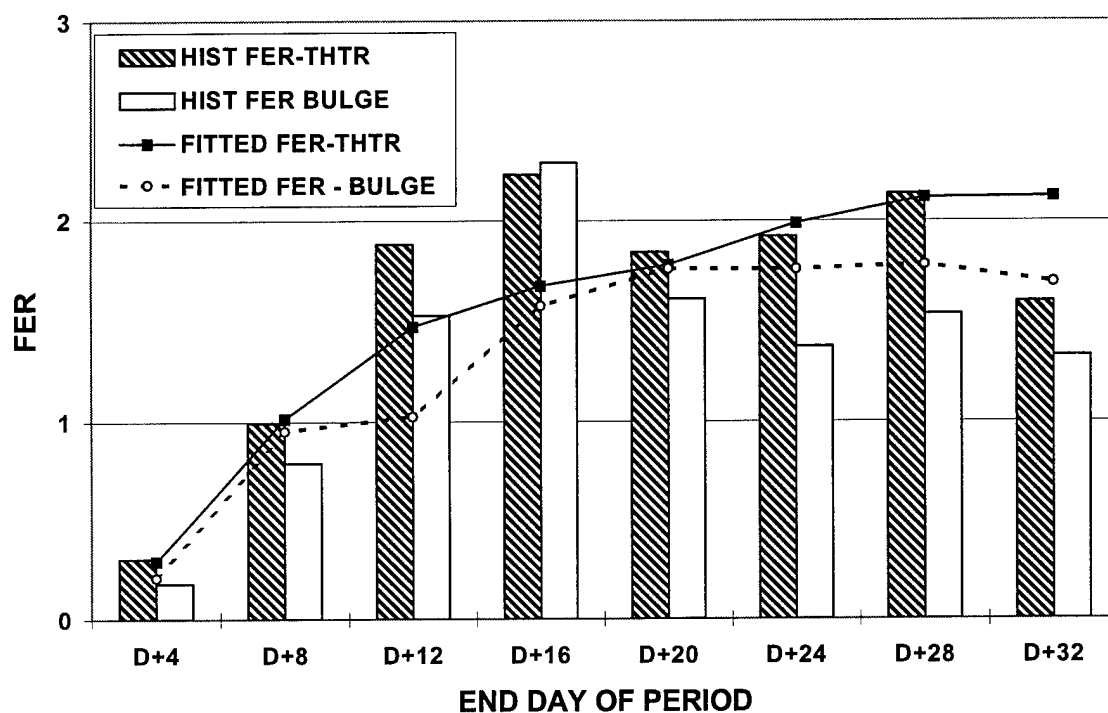


Figure E-6. Historical Personnel FER vs Exponential Fitted FER (combat casualties)

Table E-6. Historical Personnel FER vs Exponential Fitted FER (combat casualties)

	D+4	D+8	D+12	D+16	D+20	D+24	D+28	D+32
HIST FER-THTR	0.312	0.993	1.88	2.235	1.84	1.919	2.137	1.592
FITTED FER-THTR	0.30	1.01	1.47	1.67	1.78	1.99	2.12	2.12
HIST FER BULGE	0.184	0.793	1.525	2.295	1.606	1.37	1.53	1.324
FITTED FER - BULGE	0.21	0.95	1.02	1.57	1.76	1.75	1.78	1.69

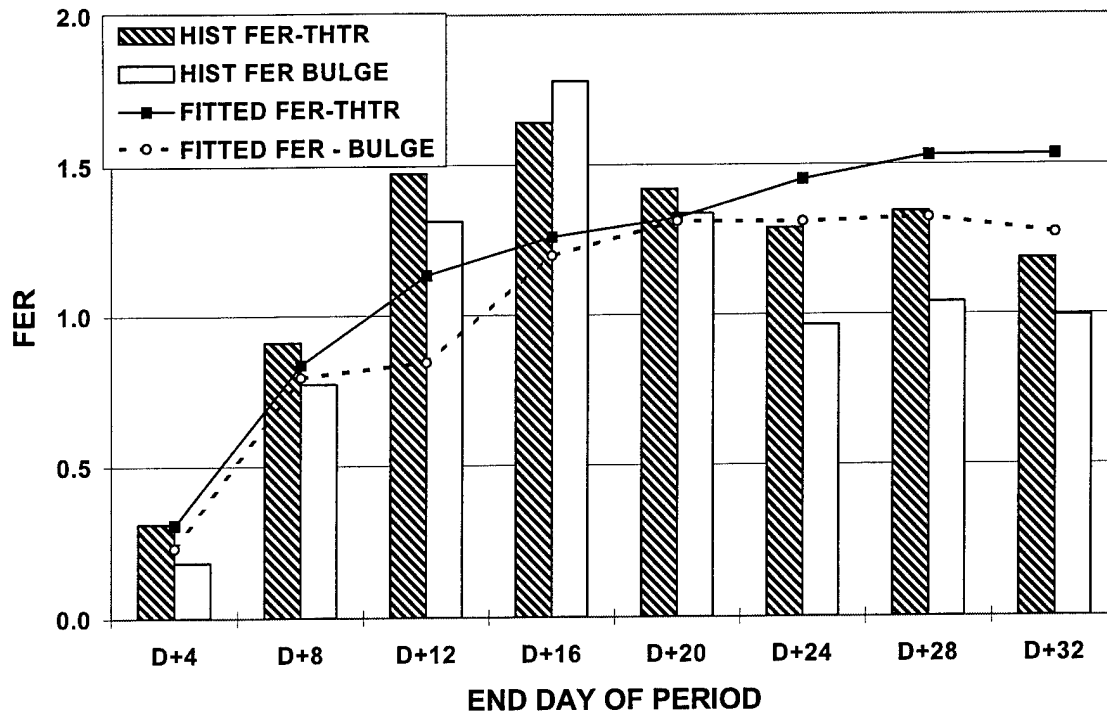
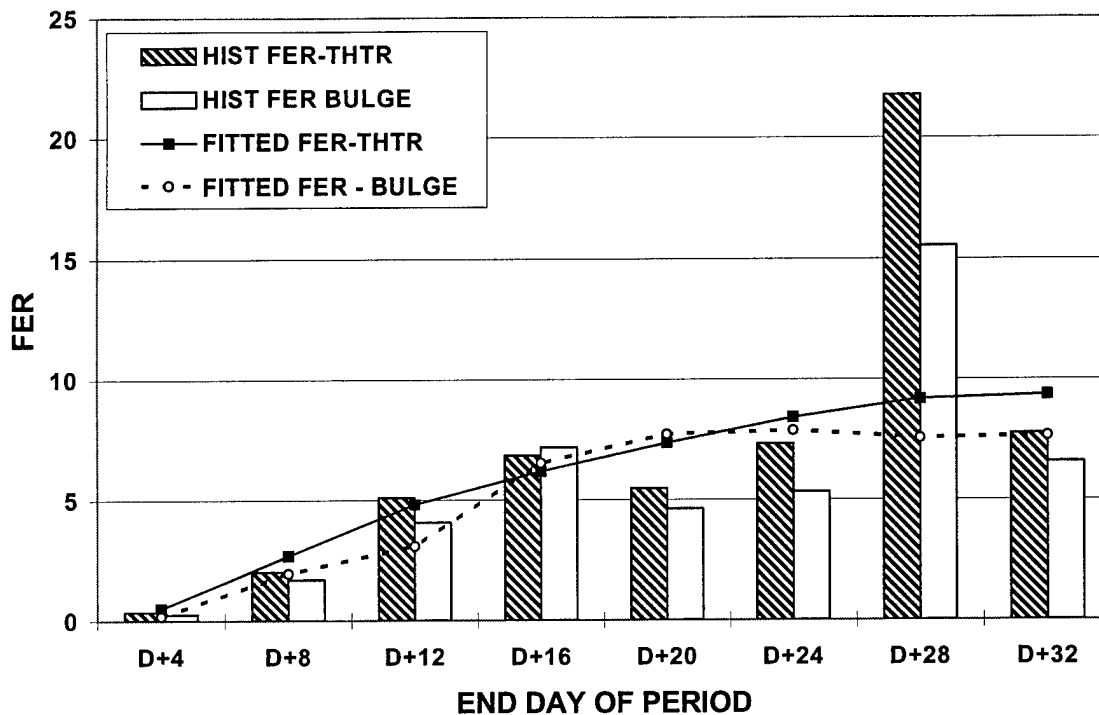


Figure E-7. Historical Personnel FER vs Exponential Fitted FER (total casualties)

Table E-7. Historical Personnel FER vs Exponential Fitted FER
(total casualties)

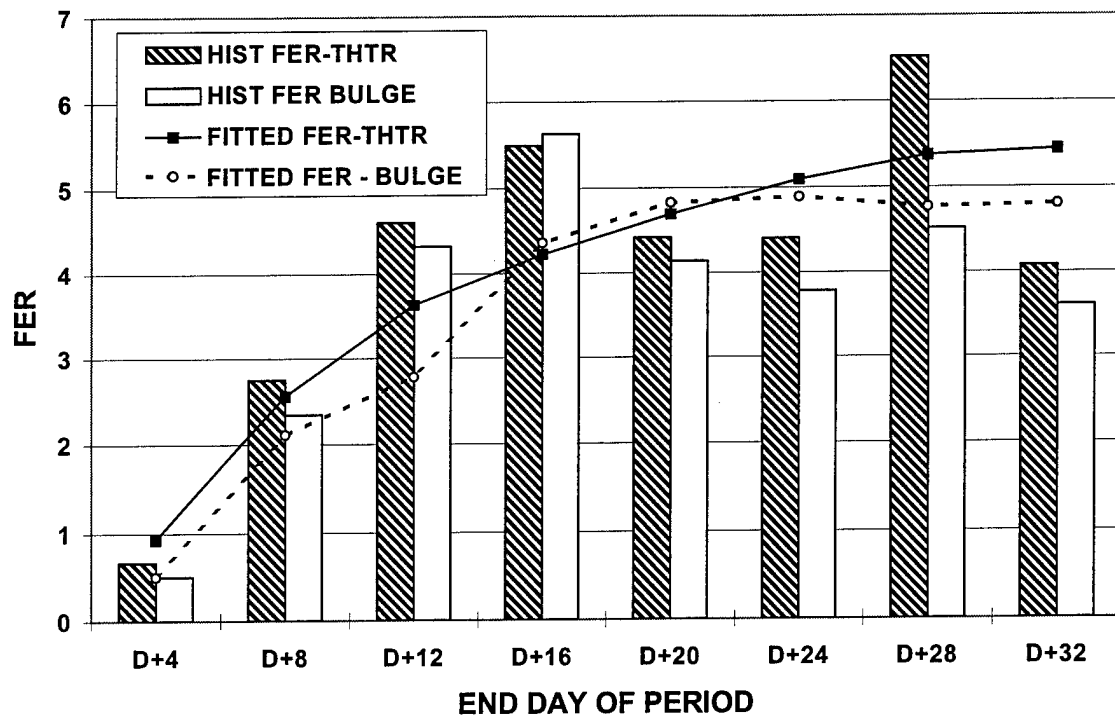
	D+4	D+8	D+12	D+16	D+20	D+24	D+28	D+32
HIST FER-THTR	0.312	0.909	1.472	1.64	1.418	1.289	1.344	1.184
FITTED FER-THTR	0.31	0.84	1.13	1.26	1.32	1.45	1.53	1.53
HIST FER BULGE	0.185	0.772	1.314	1.778	1.338	0.964	1.04	0.993
FITTED FER - BULGE	0.23	0.79	0.84	1.20	1.31	1.31	1.32	1.27



**Figure E-8. Historical Personnel FER vs Exponential Fitted FER
(all destroyed or abandoned major systems)**

**Table E-8. Historical Personnel FER vs Exponential Fitted FER
(all destroyed or abandoned major systems)**

	D+4	D+8	D+12	D+16	D+20	D+24	D+28	D+32
HIST FER-THTR	0.37	2.04	5.08	6.82	5.46	7.29	21.77	7.72
FITTED FER-THTR	0.51	2.70	4.81	6.17	7.33	8.39	9.17	9.36
HIST FER BULGE	0.26	1.72	4.06	7.15	4.63	5.32	15.54	6.54
FITTED FER - BULGE	0.19	1.97	3.10	6.50	7.69	7.85	7.53	7.62



**Figure E-9. Historical Personnel FER vs Exponential Fitted FER
(all damaged, destroyed and abandoned major systems)**

**Table E-9. Historical Personnel FER vs Exponential Fitted FER
(all damaged, destroyed, or abandoned major systems)**

	D+4	D+8	D+12	D+16	D+20	D+24	D+28	D+32
HIST FER-THTR	0.67	2.748	4.607	5.493	4.417	4.399	6.513	4.075
FITTED FER-THTR	0.93	2.56	3.63	4.22	4.69	5.09	5.37	5.44
HIST FER BULGE	0.509	2.339	4.323	5.631	4.136	3.78	4.512	3.604
FITTED FER - BULGE	0.51	2.11	2.78	4.36	4.83	4.89	4.77	4.80

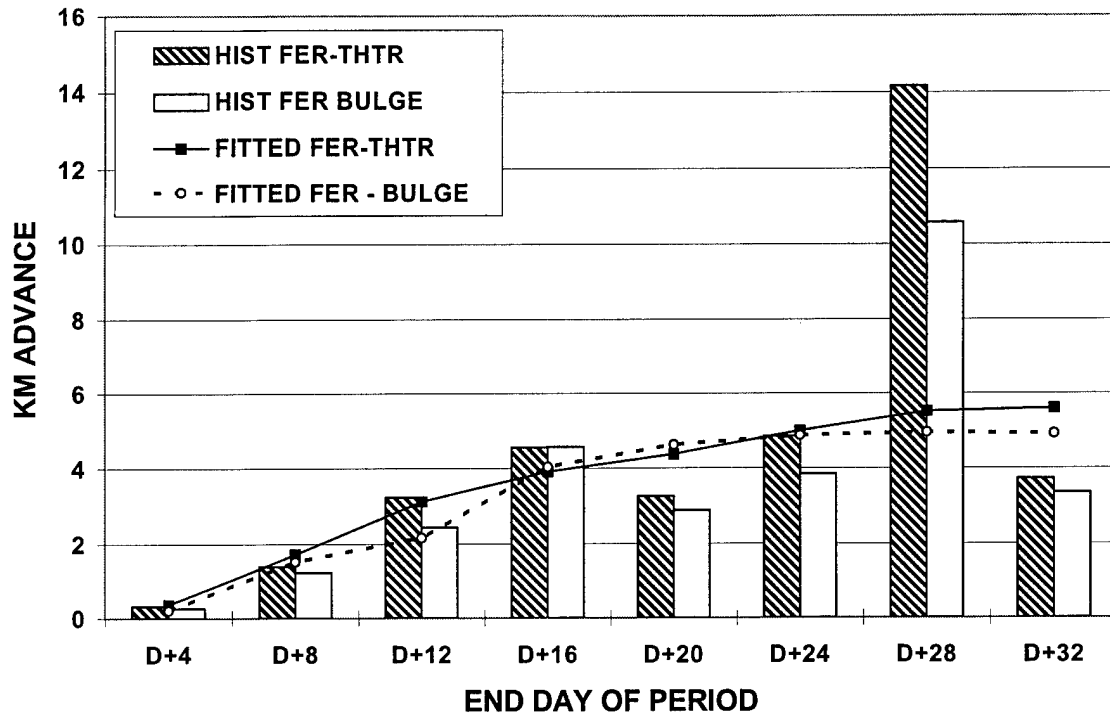
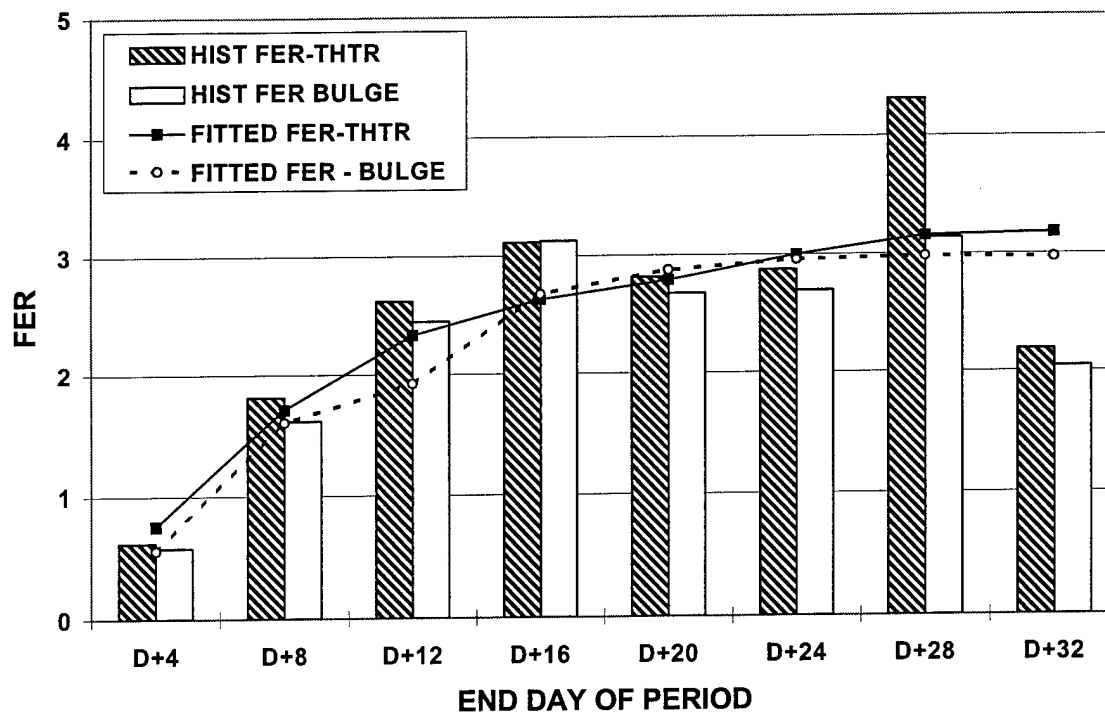


Figure E-10. Historical Personnel FER vs Exponential Fitted FER
(all destroyed and abandoned tanks, AT/Ms, artillery)

Table E-10. Historical Personnel FER vs Exponential Fitted FER
(all destroyed or abandoned tanks, AT/Ms, artillery)

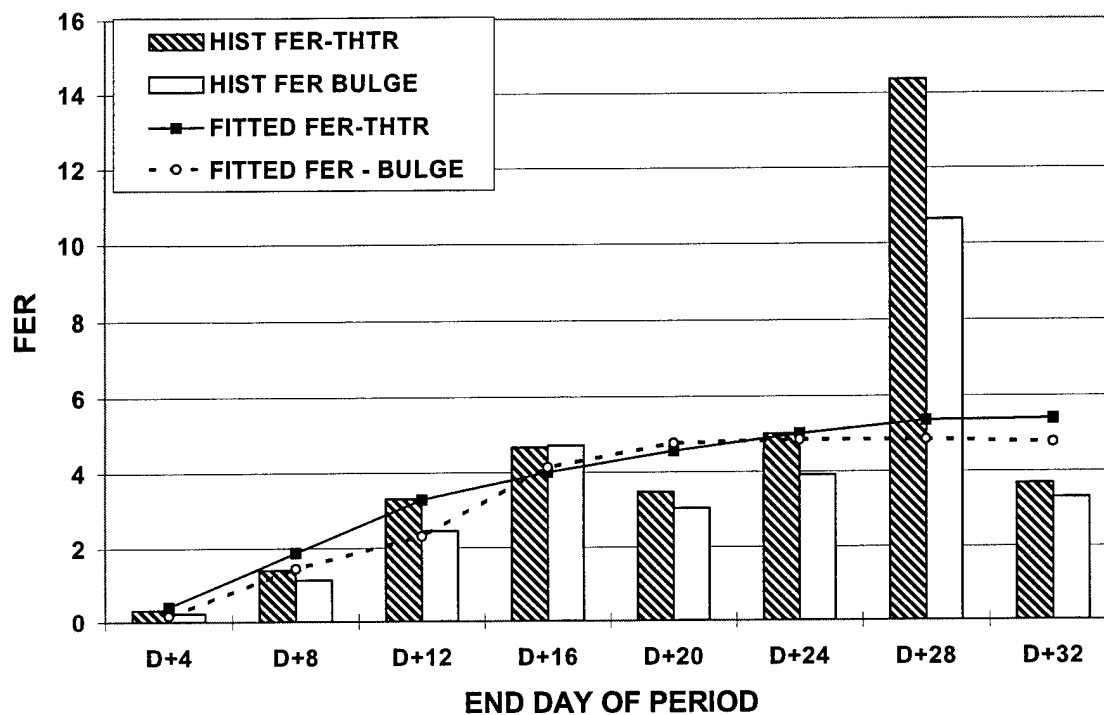
	D+4	D+8	D+12	D+16	D+20	D+24	D+28	D+32
HIST FER-THTR	0.327	1.403	3.229	4.554	3.261	4.854	14.159	3.719
FITTED FER-THTR	0.37	1.72	3.12	3.91	4.37	5.00	5.52	5.60
HIST FER BULGE	0.265	1.233	2.438	4.573	2.875	3.847	10.576	3.356
FITTED FER - BULGE	0.21	1.53	2.16	4.03	4.62	4.87	4.95	4.92



**Figure E-11. Historical Personnel FER vs Exponential Fitted FER
(all damaged, destroyed, and abandoned tanks, AT/Ms, artillery)**

**Table E-11. Historical Personnel FER vs Exponential Fitted FER
(all damaged, destroyed, or abandoned tanks, AT/Ms, artillery)**

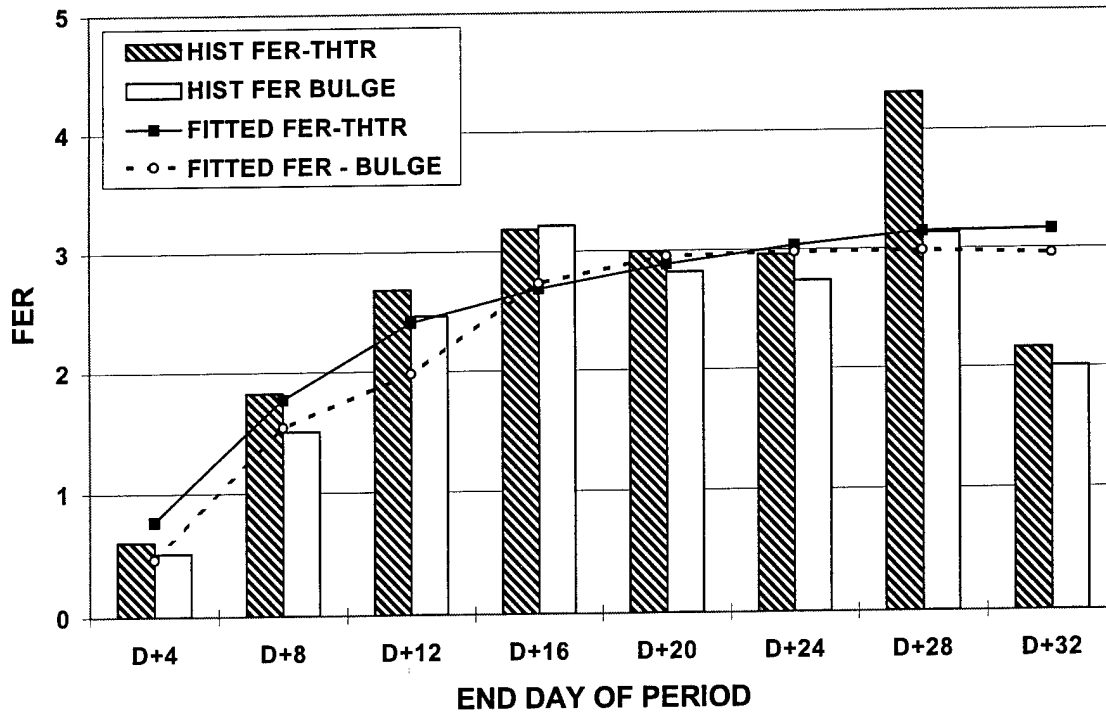
	D+4	D+8	D+12	D+16	D+20	D+24	D+28	D+32
HIST FER-THTR	0.622	1.81	2.618	3.111	2.813	2.871	4.306	2.184
FITTED FER-THTR	0.76	1.71	2.33	2.63	2.79	2.99	3.15	3.18
HIST FER BULGE	0.581	1.611	2.446	3.125	2.673	2.693	3.135	2.04
FITTED FER - BULGE	0.56	1.60	1.92	2.67	2.87	2.95	2.98	2.97



**Figure E-12. Historical Personnel FER vs Exponential Fitted FER
(all destroyed and abandoned tanks, AT wpns, artillery)**

**Table E-12. Historical Personnel FER vs Exponential Fitted FER
(all destroyed or abandoned tanks, AT wpns, artillery)**

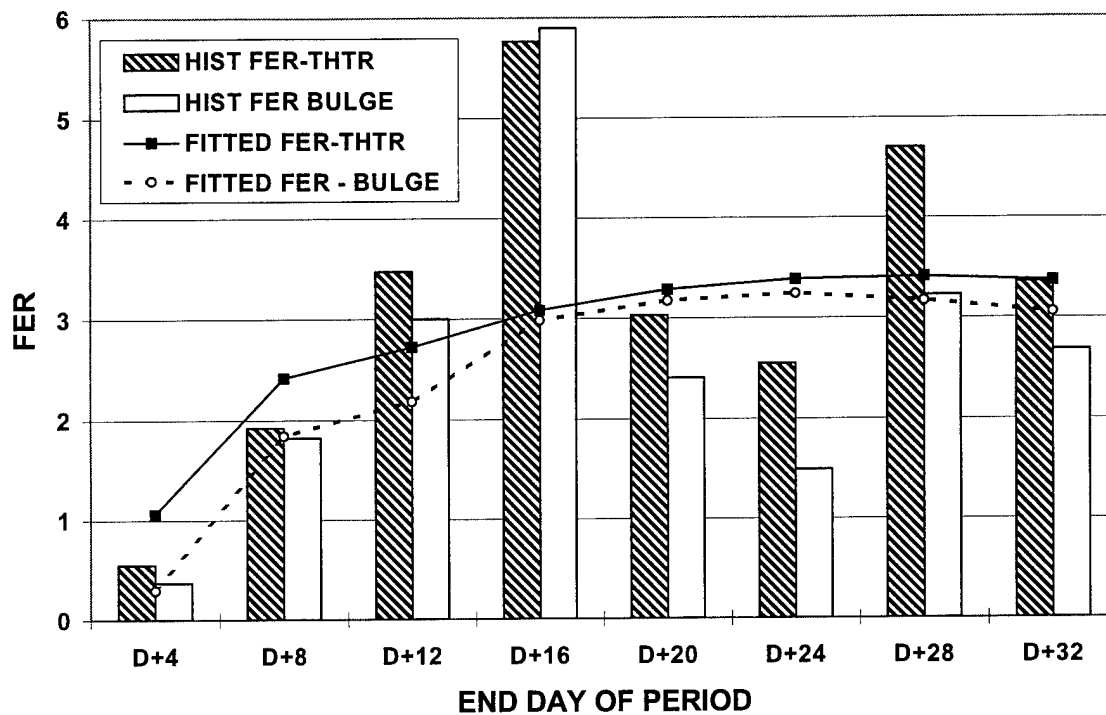
	D+4	D+8	D+12	D+16	D+20	D+24	D+28	D+32
HIST FER-THTR	0.32	1.41	3.31	4.67	3.47	5.02	14.36	3.69
FITTED FER-THTR	0.41	1.88	3.28	4.01	4.57	5.01	5.36	5.40
HIST FER BULGE	0.23	1.15	2.45	4.72	3.03	3.92	10.64	3.31
FITTED FER - BULGE	0.16	1.46	2.30	4.14	4.77	4.85	4.86	4.78



**Figure E-13. Historical Personnel FER vs Exponential Fitted FER
(all damaged, destroyed and abandoned tanks, AT wpns, artillery)**

**Table E-13. Historical Personnel FER vs Exponential Fitted FER
(all damaged, destroyed, or abandoned tanks, AT wpns, artillery)**

	D+4	D+8	D+12	D+16	D+20	D+24	D+28	D+32
HIST FER-THTR	0.60	1.82	2.68	3.18	2.99	2.96	4.31	2.16
FITTED FER-THTR	0.77	1.77	2.40	2.68	2.88	3.03	3.15	3.16
HIST FER BULGE	0.513	1.5	2.461	3.215	2.817	2.737	3.132	2.007
FITTED FER - BULGE	0.47	1.54	1.98	2.73	2.95	2.98	2.98	2.95



**Figure E-14. Historical Personnel FER vs Exponential Fitted FER
(all destroyed and abandoned tanks)**

**Table E-14. Historical Personnel FER vs Exponential Fitted FER
(all destroyed or abandoned tanks)**

	D+4	D+8	D+12	D+16	D+20	D+24	D+28	D+32
HIST FER-THTR	0.557	1.919	3.469	5.764	3.029	2.549	4.702	3.356
FITTED FER-THTR	1.06	2.42	2.72	3.08	3.29	3.39	3.41	3.37
HIST FER BULGE	0.374	1.816	2.999	5.896	2.406	1.485	3.225	2.685
FITTED FER - BULGE	0.30	1.84	2.18	2.98	3.17	3.24	3.17	3.06

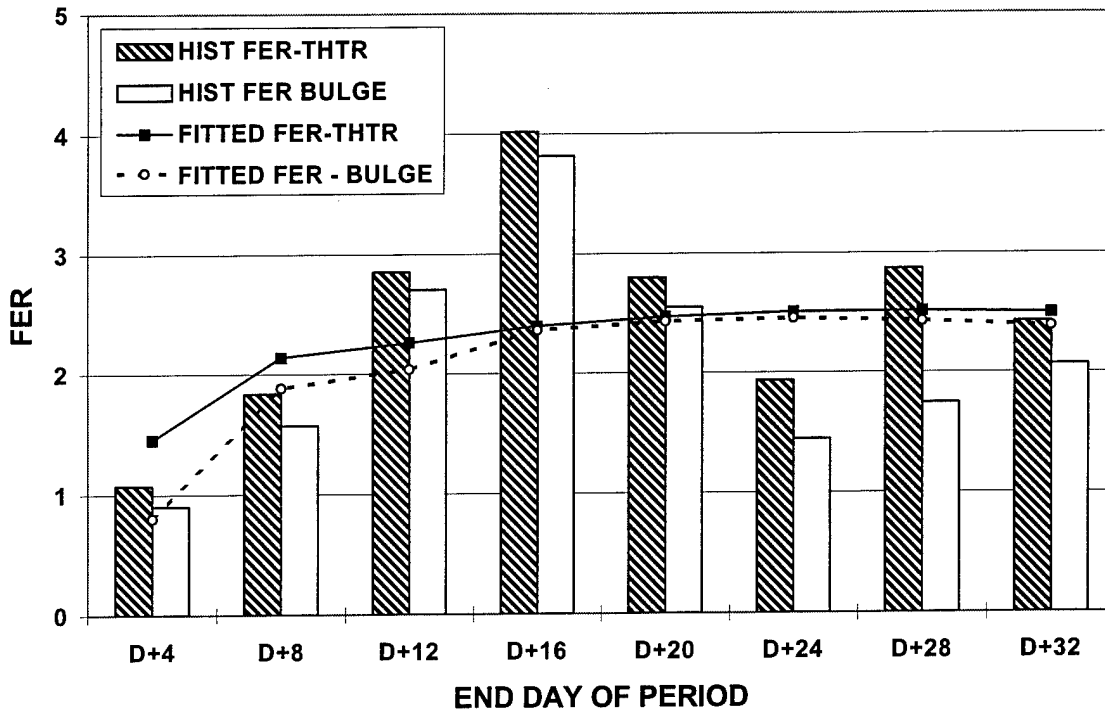


Figure E-15. Historical Personnel FER vs Exponential Fitted FER
(all damaged, destroyed, and abandoned tanks)

Table E-15. Historical Personnel FER vs Exponential Fitted FER
(all damaged, destroyed, or abandoned tanks)

	D+4	D+8	D+12	D+16	D+20	D+24	D+28	D+32
HIST FER-THTR	1.07	1.833	2.853	4.017	2.799	1.935	2.867	2.428
FITTED FER-THTR	1.45	2.14	2.26	2.40	2.47	2.51	2.51	2.50
HIST FER BULGE	0.899	1.569	2.704	3.814	2.55	1.445	1.744	2.068
FITTED FER - BULGE	0.80	1.88	2.04	2.36	2.43	2.45	2.43	2.39

APPENDIX F

LINEAR TRENDS BETWEEN ADVANTAGE FACTOR AND FEBA MOVEMENT

OVERVIEW. This appendix supplements Chapter 5 of the report.

a. Figures F-1 through F-5 show scatter plots with linear trend lines for advantage factor (ADV) versus FEBA advance, as computed from ACSDB data. Tables F-1 through F-5 tabulate the data plotted in the figures. ADV is defined favoring the Germans and is approximated as $ADV = -.5 \ln(FER \text{ favoring US/UK})$, where FER is computed from the ACSDB for each of the 10 cases associated with the data tabulated in Table 5-1. Figures F-1 and F-2 appear in Chapter 5 as Figures 5-2 and 5-1, respectively. Each scatter plot case consists of a set of 16 ADV values plotted against 16 associated FEBA advances, for 4-day periods during the campaign. FEBA advance during each 4-day period is expressed in km. Eight of the ADV values in each plot are computed for the full ARCAS theater, and eight are computed only for the ARCAS bulge. Each scatter plot case corresponds to one of the 10 FER cases described in Table 2-1. Each scatter plot also shows the fitted linear trend line, the fitted equation, and the value of R^2 , the coefficient of determination, for the trend line. Each figure shows two scatter plots, which differ only in the damage criterion used to compute the FER for the plotted ADV values. All 32 plotted points in each figure have the ADV values based on the same mix of systems (or personnel). The system mix is identified in the figure label. Each set of plotted points and trend lines in each figure is labeled with its associated damage criterion.

b. Figures F-6 through F-15 show the linear-form fitted FEBA movement, plotted against the historical (ACSDB) FEBA movement, for each of the linear fits in the 10 cases with characteristics tabulated in Table 5-1. Tables F-6 through F-15 tabulate the data plotted in the figures. Each fitted FEBA movement is determined by the appropriate fitted equation for that case, using the historical ADV value for that period and case.

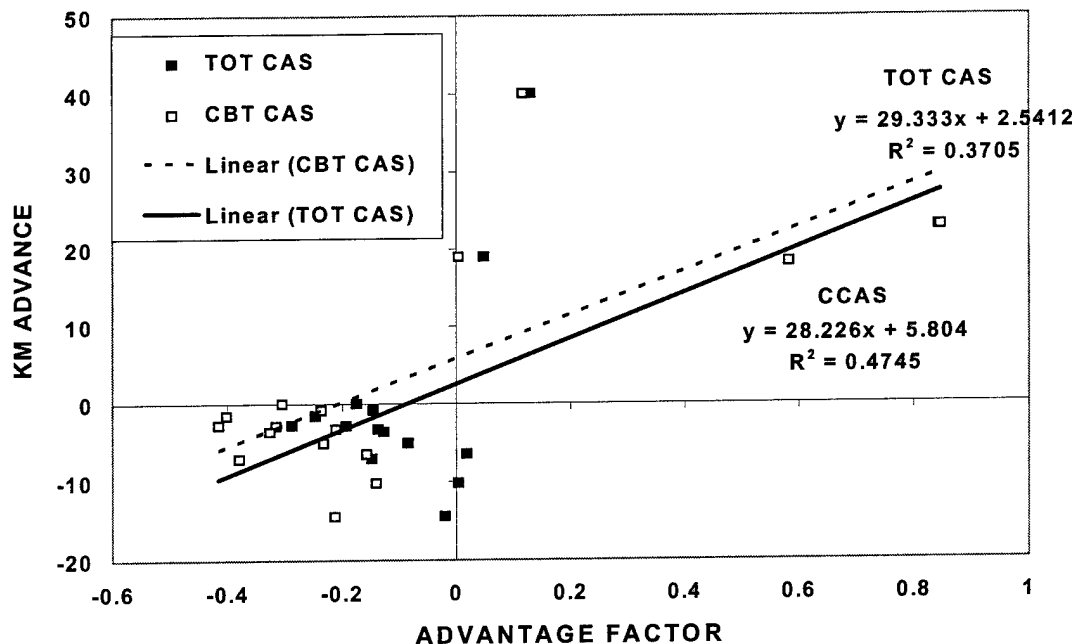


Figure F-1. Linear Fit of Personnel Advantage Factor vs FEBA Movement

Table F-1. Linear Fit of Personnel Advantage Factor vs FEBA Movement

		D+4	D+8	D+12	D+16	D+20	D+24	D+28	D+32
TOT CAS	ADV FAC - THTR	0.58	0.05	-0.19	-0.25	-0.17	-0.13	-0.15	-0.08
	ADV FAC - BULGE	0.84	0.13	-0.14	-0.29	-0.15	0.02	-0.02	0.00
	KM ADV - THTR	18.24	18.96	-2.83	-1.5	0.03	-3.56	-7.01	-5.02
	KM ADV - BULGE	22.83	40.08	-3.23	-2.72	-0.8	-6.45	-14.41	-10.16
CBT CAS	ADV FAC - THTR	0.58	0.00	-0.32	-0.40	-0.30	-0.33	-0.38	-0.23
	ADV FAC - BULGE	0.85	0.12	-0.21	-0.42	-0.24	-0.16	-0.21	-0.14
	KM ADV - THTR	18.24	18.96	-2.83	-1.5	0.03	-3.56	-7.01	-5.02
	KM ADV - BULGE	22.83	40.08	-3.23	-2.72	-0.8	-6.45	-14.41	-10.16

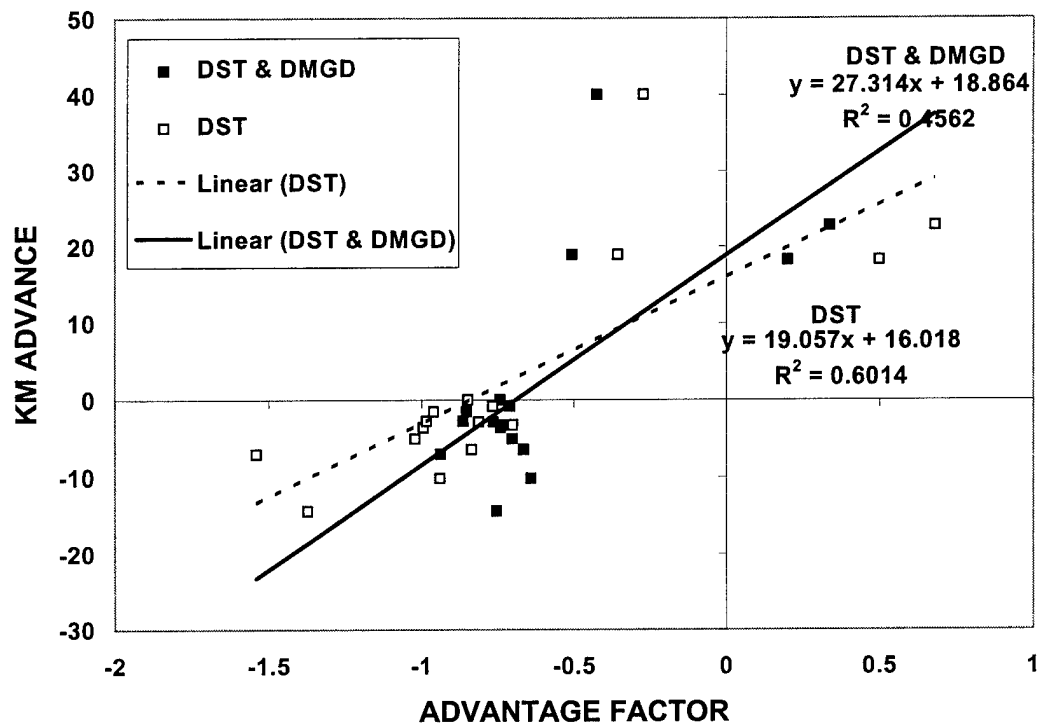


Figure F-2. Linear Fit of System Advantage Factor vs FEBA Movement
(all major systems)

Table F-2. Linear Fit of System Advantage Factor vs FEBA Movement (all major systems)

		D+4	D+8	D+12	D+16	D+20	D+24	D+28	D+32
DST & DMGD	ADV FAC - THTR	0.20	-0.51	-0.76	-0.85	-0.74	-0.74	-0.94	-0.70
	ADV FAC - BULGE	0.34	-0.42	-0.73	-0.86	-0.71	-0.66	-0.75	-0.64
	KM ADV - THTR	18.24	18.96	-2.83	-1.5	0.03	-3.56	-7.01	-5.02
	KM ADV - BULGE	22.83	40.08	-3.23	-2.72	-0.8	-6.45	-14.41	-10.16
DST	ADV FAC - THTR	0.50	-0.36	-0.81	-0.96	-0.85	-0.99	-1.54	-1.02
	ADV FAC - BULGE	0.68	-0.27	-0.70	-0.98	-0.77	-0.84	-1.37	-0.94
	KM ADV - THTR	18.24	18.96	-2.83	-1.5	0.03	-3.56	-7.01	-5.02
	KM ADV - BULGE	22.83	40.08	-3.23	-2.72	-0.8	-6.45	-14.41	-10.16

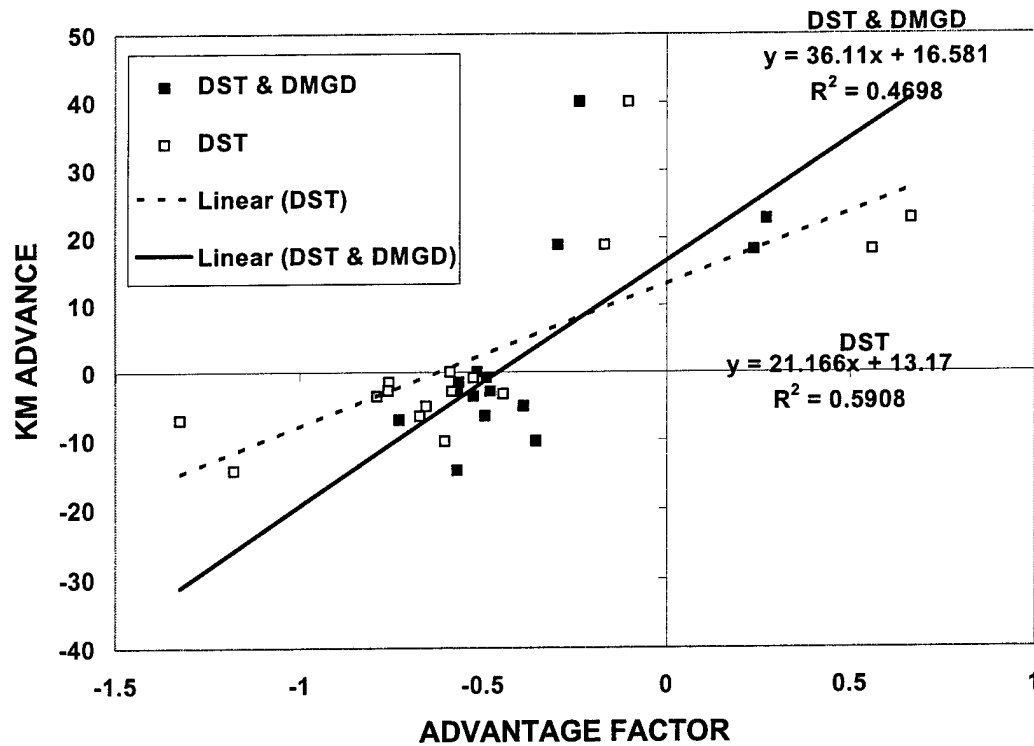


Figure F-3. Linear Fit of System Advantage Factor vs FEBA Movement (tanks, AT/Ms, artillery)

Table F-3. Linear Fit of System Advantage Factor vs FEBA Movement (tanks, AT/Ms, artillery)

		D+4	D+8	D+12	D+16	D+20	D+24	D+28	D+32
DST & DMGD	ADV FAC - THTR	0.24	-0.30	-0.48	-0.57	-0.52	-0.53	-0.73	-0.39
	ADV FAC - BULGE	0.27	-0.24	-0.45	-0.57	-0.49	-0.50	-0.57	-0.36
	KM ADV - THTR	18.24	18.96	-2.83	-1.5	0.03	-3.56	-7.01	-5.02
	KM ADV - BULGE	22.83	40.08	-3.23	-2.72	-0.8	-6.45	-14.41	-10.16
DST	ADV FAC - THTR	0.56	-0.17	-0.59	-0.76	-0.59	-0.79	-1.33	-0.66
	ADV FAC - BULGE	0.66	-0.10	-0.45	-0.76	-0.53	-0.67	-1.18	-0.61
	KM ADV - THTR	18.24	18.96	-2.83	-1.5	0.03	-3.56	-7.01	-5.02
	KM ADV - BULGE	22.83	40.08	-3.23	-2.72	-0.8	-6.45	-14.41	-10.16

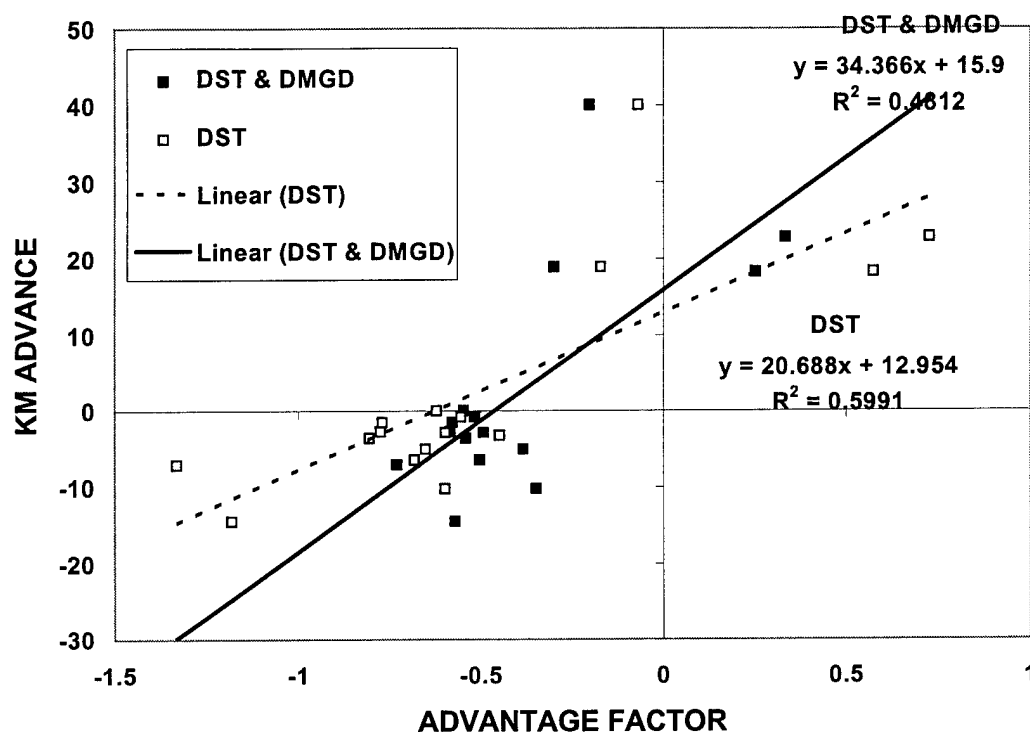


Figure F-4. Linear Fit of System Advantage Factor vs FEBA Movement (tanks, AT weapons, artillery)

Table F-4. Linear Fit of System Advantage Factor vs FEBA Movement (tanks, AT wpns, artillery)

		D+4	D+8	D+12	D+16	D+20	D+24	D+28	D+32
DST & DMGD	ADV FAC - THTR	0.25	-0.30	-0.49	-0.58	-0.55	-0.54	-0.73	-0.38
	ADV FAC - BULGE	0.33	-0.20	-0.45	-0.58	-0.52	-0.50	-0.57	-0.35
	KM ADV - THTR	18.24	18.96	-2.83	-1.5	0.03	-3.56	-7.01	-5.02
	KM ADV - BULGE	22.83	40.08	-3.23	-2.72	-0.8	-6.45	-14.41	-10.16
DST	ADV FAC - THTR	0.57	-0.17	-0.60	-0.77	-0.62	-0.81	-1.33	-0.65
	ADV FAC - BULGE	0.73	-0.07	-0.45	-0.78	-0.55	-0.68	-1.18	-0.60
	KM ADV - THTR	18.24	18.96	-2.83	-1.5	0.03	-3.56	-7.01	-5.02
	KM ADV - BULGE	22.83	40.08	-3.23	-2.72	-0.8	-6.45	-14.41	-10.16

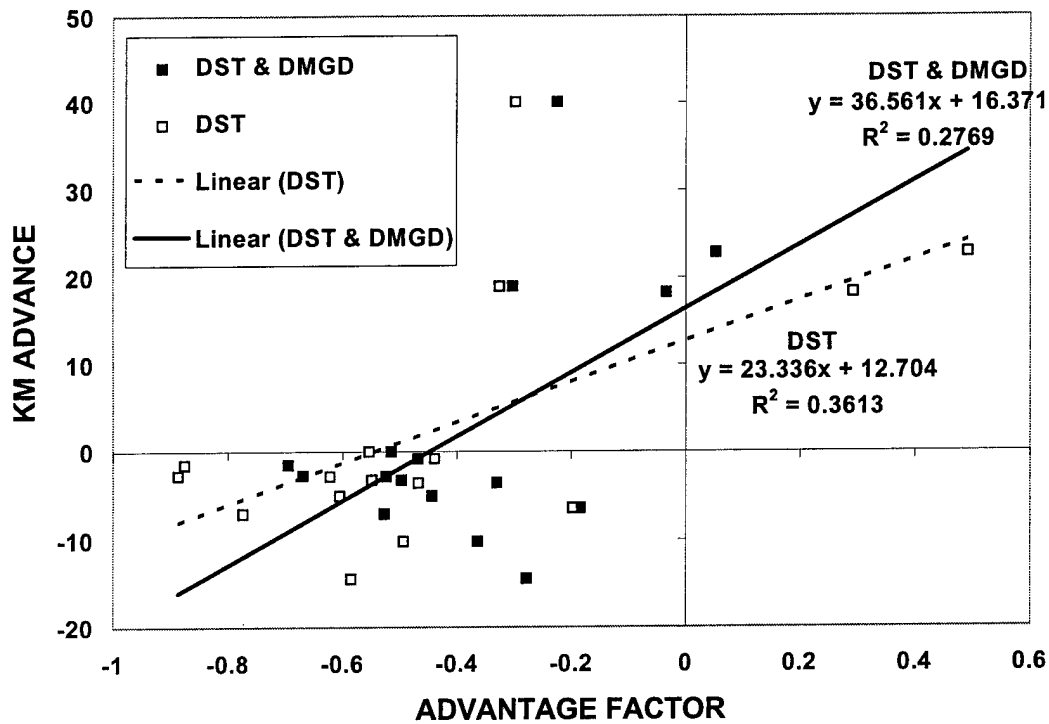


Figure F-5. Linear Fit of System Advantage Factor vs FEBA Movement (tanks only)

Table F-5. Linear Fit of System Advantage Factor vs FEBA Movement (tanks only)

		D+4	D+8	D+12	D+16	D+20	D+24	D+28	D+32
DST & DMGD	ADV FAC - THTR	-0.03	-0.30	-0.52	-0.70	-0.51	-0.33	-0.53	-0.44
	ADV FAC - BULGE	0.05	-0.23	-0.50	-0.67	-0.47	-0.18	-0.28	-0.36
	KM ADV - THTR	18.24	18.96	-2.83	-1.5	0.03	-3.56	-7.01	-5.02
	KM ADV - BULGE	22.83	40.08	-3.23	-2.72	-0.8	-6.45	-14.41	-10.16
DST	ADV FAC - THTR	0.29	-0.33	-0.62	-0.88	-0.55	-0.47	-0.77	-0.61
	ADV FAC - BULGE	0.49	-0.30	-0.55	-0.89	-0.44	-0.20	-0.59	-0.49
	KM ADV - THTR	18.24	18.96	-2.83	-1.5	0.03	-3.56	-7.01	-5.02
	KM ADV - BULGE	22.83	40.08	-3.23	-2.72	-0.8	-6.45	-14.41	-10.16

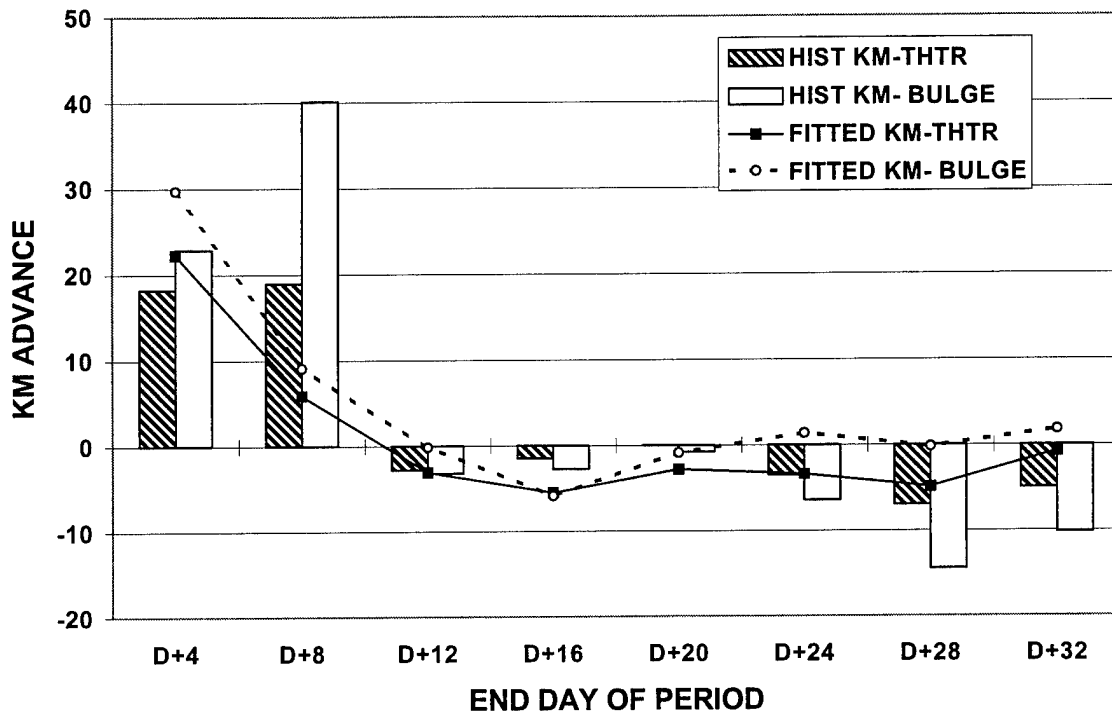
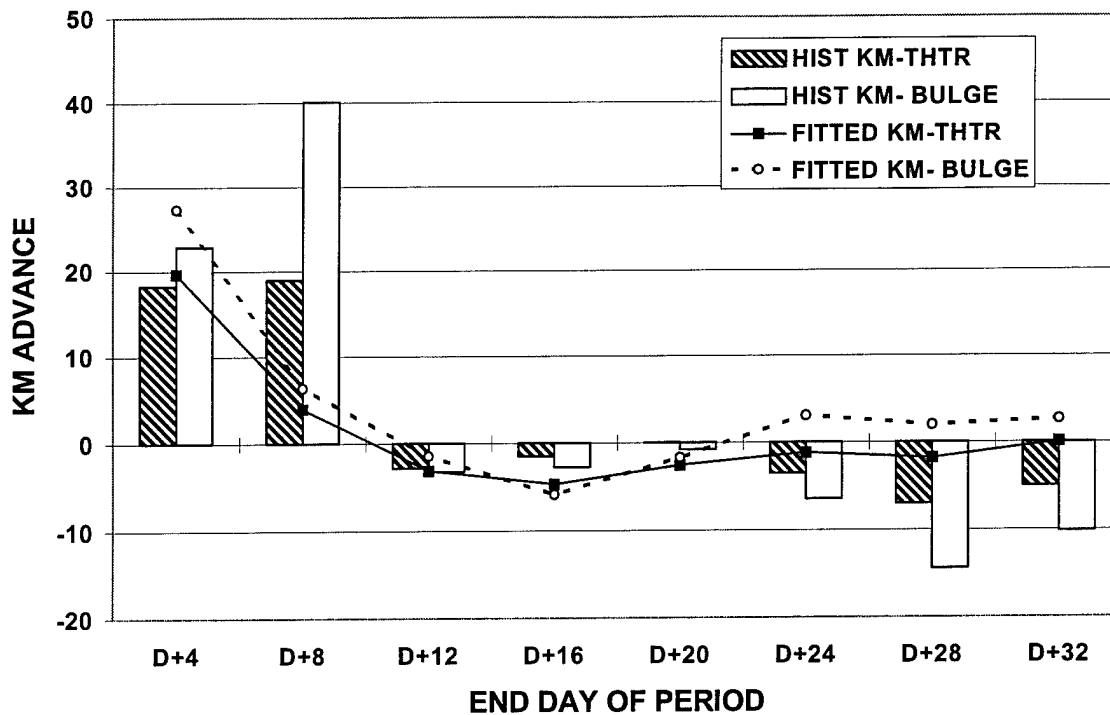


Figure F-6. Historical FEBA Movement vs Linearly Fitted FEBA Movement
(personnel ADV using combat casualties)

Table F-6. Historical FEBA Movement vs Linearly Fitted FEBA Movement
(personnel ADV using combat casualties)

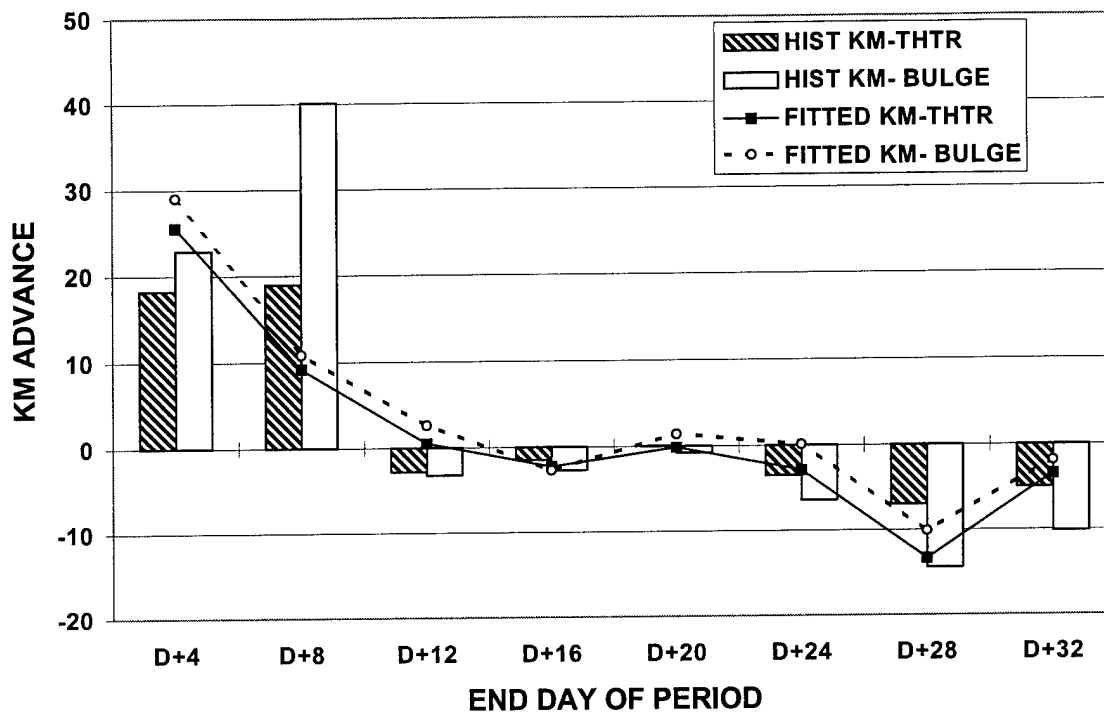
	D+4	D+8	D+12	D+16	D+20	D+24	D+28	D+32
HIST KM-THTR	48.24	18.96	-2.83	-1.5	0.03	-3.56	-7.01	-5.02
FITTED KM-THTR	22.26	5.90	-3.12	-5.56	-2.82	-3.41	-4.93	-0.77
HIST KM- BULGE	22.83	40.08	-3.23	-2.72	-0.8	-6.45	-14.41	-10.16
FITTED KM- BULGE	29.72	9.08	-0.16	-5.94	-0.89	1.35	-0.21	1.83



**Figure F-7. Historical FEBA Movement vs Linearly Fitted FEBA Movement
(personnel ADV using total casualties)**

**Table F-7. Historical FEBA Movement vs Linearly Fitted FEBA Movement
(personnel ADV using total casualties)**

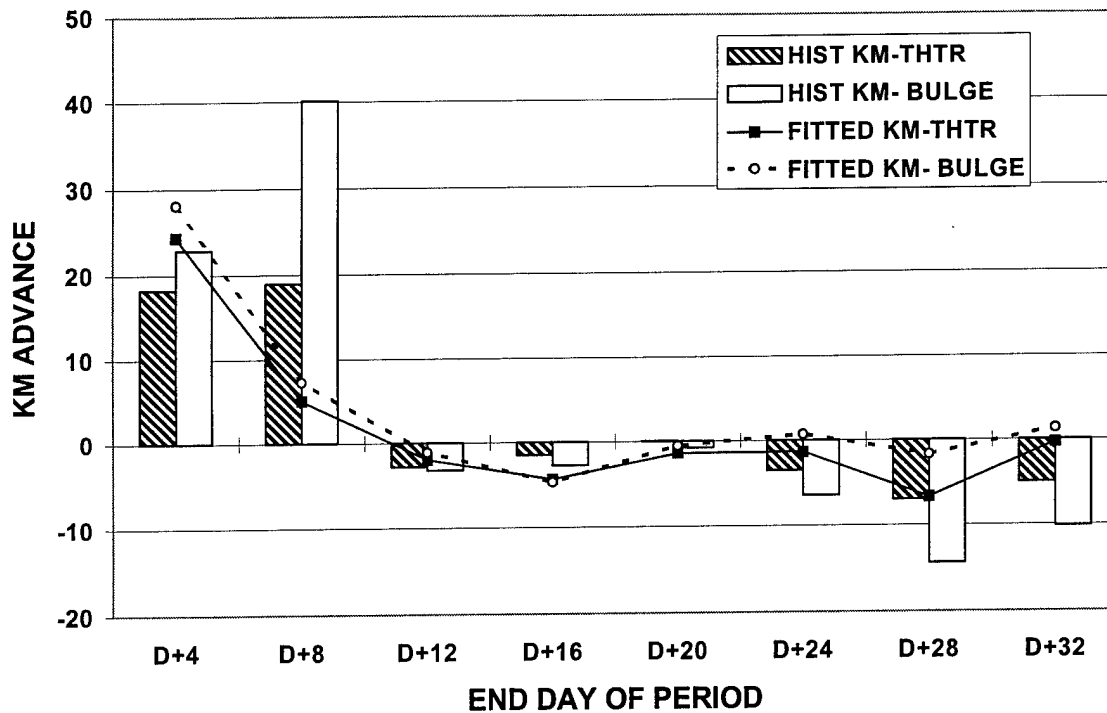
	D+4	D+8	D+12	D+16	D+20	D+24	D+28	D+32
HIST KM-THTR	18.24	18.96	-2.83	-1.5	0.03	-3.56	-7.01	-5.02
FITTED KM-THTR	19.62	3.94	-3.13	-4.71	-2.58	-1.18	-1.80	0.06
HIST KM- BULGE	22.83	40.08	-3.23	-2.72	-0.8	-6.45	-14.41	-10.16
FITTED KM- BULGE	27.29	6.33	-1.46	-5.90	-1.73	3.08	1.96	2.64



**Figure F-8. Historical FEBA Movement vs Linearly Fitted FEBA Movement
(all destroyed and abandoned major systems)**

**Table F-8. Historical FEBA Movement vs Linearly Fitted FEBA Movement
(all destroyed and abandoned major systems)**

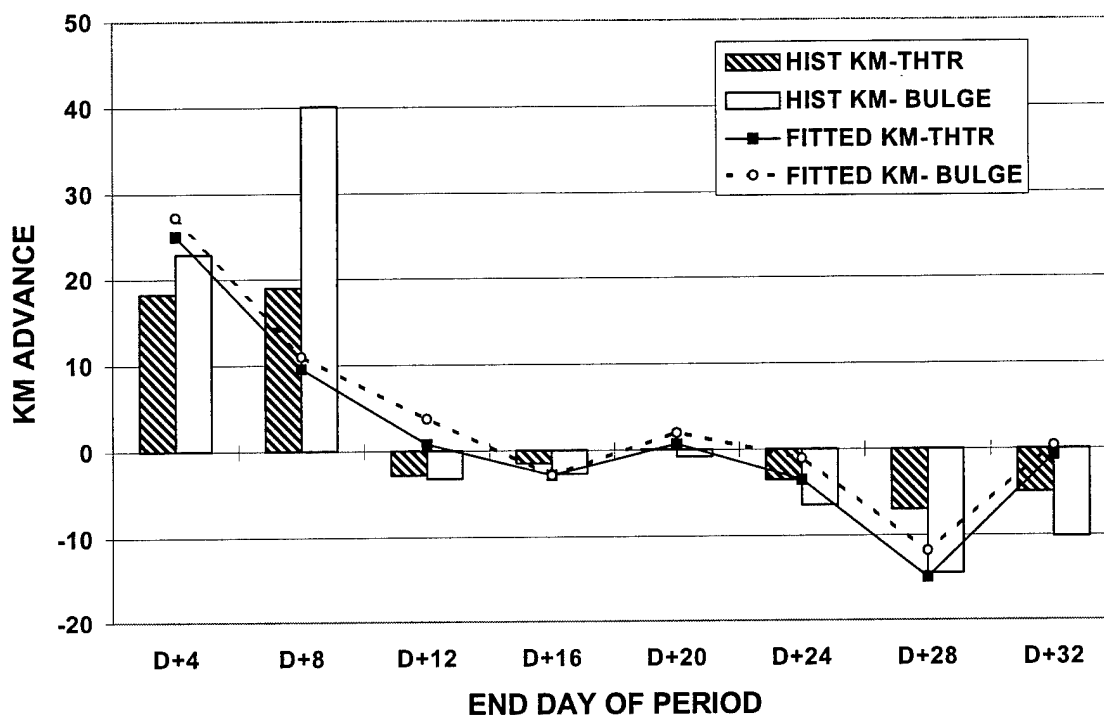
	D+4	D+8	D+12	D+16	D+20	D+24	D+28	D+32
HIST KM-THTR	18.24	18.96	-2.83	-1.5	0.03	-3.56	-7.01	-5.02
FITTED KM-THTR	25.55	9.23	0.53	-2.28	-0.15	-2.92	-13.34	-3.45
HIST KM- BULGE	22.83	40.08	-3.23	-2.72	-0.8	-6.45	-14.41	-10.16
FITTED KM- BULGE	29.01	10.84	2.66	-2.73	1.42	0.09	-10.13	-1.88



**Figure F-9. Historical FEBA Movement vs Linearly Fitted FEBA Movement
(all damaged, destroyed, and abandoned major systems)**

**Table F-9. Historical FEBA Movement vs Linearly Fitted FEBA Movement
(all damaged, destroyed, and abandoned major systems)**

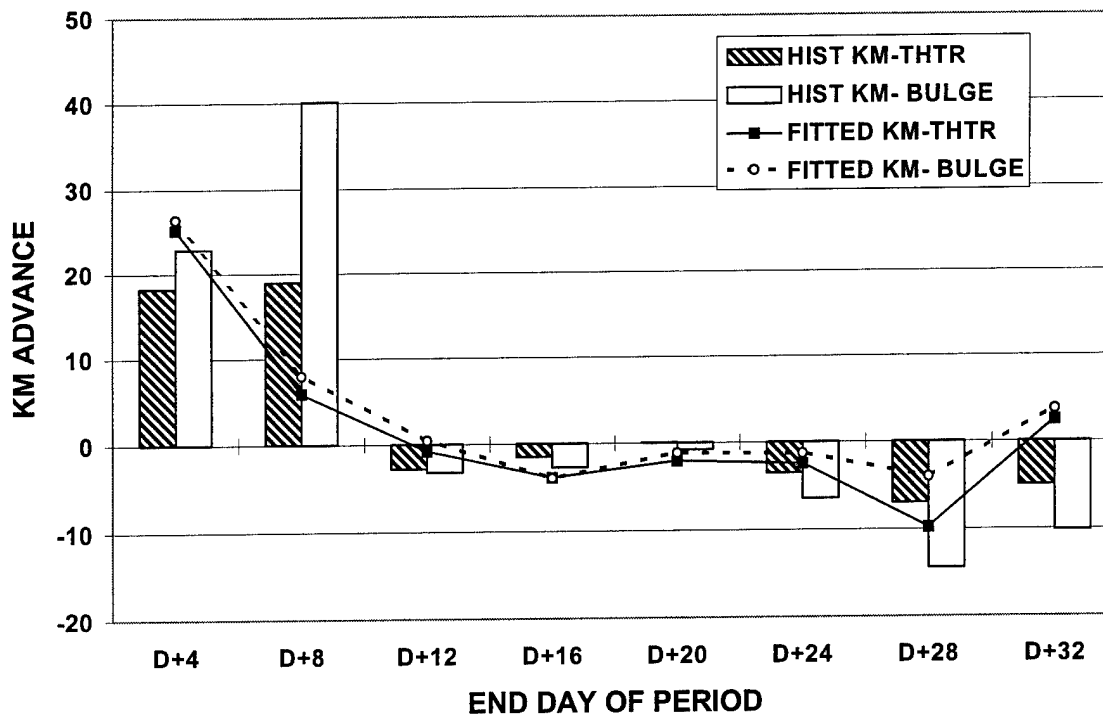
	D+4	D+8	D+12	D+16	D+20	D+24	D+28	D+32
HIST KM-THTR	18.24	18.96	-2.83	-1.5	0.03	-3.56	-7.01	-5.02
FITTED KM-THTR	24.33	5.06	-2.00	-4.40	-1.42	-1.37	-6.73	-0.32
HIST KM- BULGE	22.83	40.08	-3.23	-2.72	-0.8	-6.45	-14.41	-10.16
FITTED KM- BULGE	29.01	10.84	2.66	-2.73	1.42	0.09	-10.13	-1.88



**Figure F-10. Historical FEBA Movement vs Linearly Fitted FEBA Movement
(all destroyed and abandoned tanks, AT/Ms, artillery)**

**Table F-10. Historical FEBA Movement vs Linearly Fitted FEBA Movement
(all destroyed and abandoned tanks, AT/Ms, artillery)**

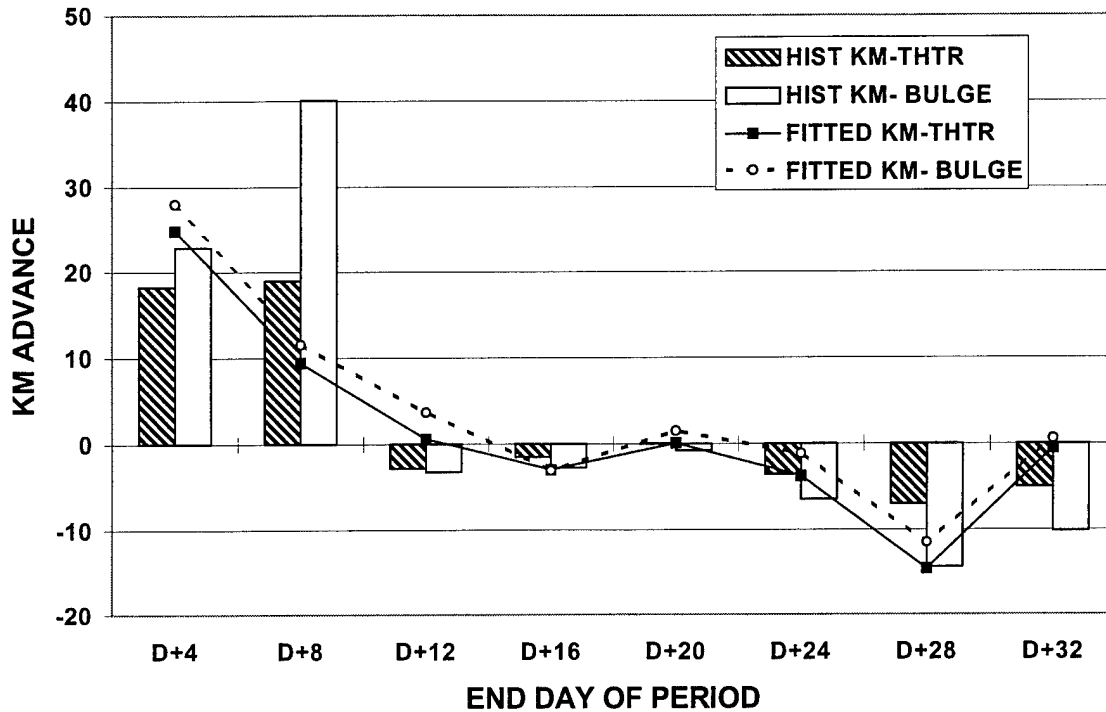
	D+4	D+8	D+12	D+16	D+20	D+24	D+28	D+32
HIST KM-THTR	18.24	18.96	-2.83	-1.5	0.03	-3.56	-7.01	-5.02
FITTED KM-THTR	25.00	9.59	0.76	-2.87	0.66	-3.55	-14.88	-0.73
HIST KM- BULGE	22.83	40.08	-3.23	-2.72	-0.8	-6.45	-14.41	-10.16
FITTED KM- BULGE	27.22	10.95	3.74	-2.92	1.99	-1.09	-11.79	0.36



**Figure F-11. Historical FEBA Movement vs Linearly Fitted FEBA Movement
(all damaged, destroyed, and abandoned tanks, AT/Ms, artillery)**

**Table F-11. Historical FEBA Movement vs Linearly Fitted FEBA Movement
(all damaged, destroyed, and abandoned tanks, AT/Ms, artillery)**

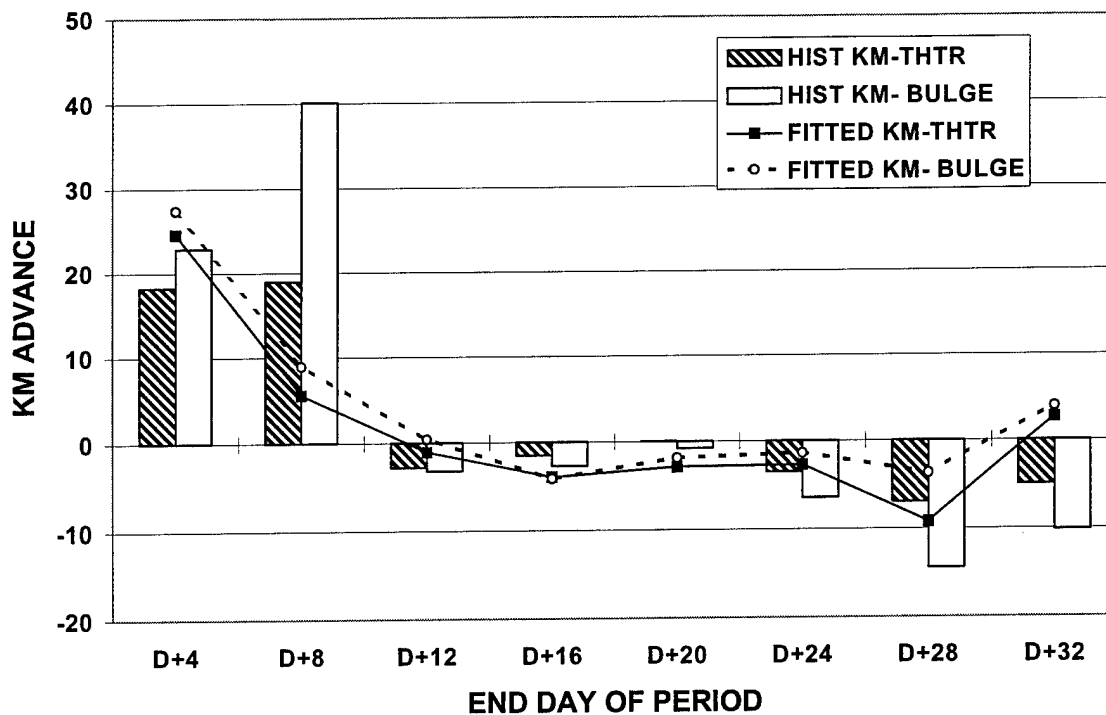
	D+4	D+8	D+12	D+16	D+20	D+24	D+28	D+32
HIST KM-THTR	18.24	18.96	-2.83	-1.5	0.03	-3.56	-7.01	-5.02
FITTED KM-THTR	25.15	5.87	-0.80	-3.91	-2.09	-2.46	-9.78	2.48
HIST KM- BULGE	22.83	40.08	-3.23	-2.72	-0.8	-6.45	-14.41	-10.16
FITTED KM- BULGE	26.38	7.97	0.43	-3.99	-1.17	-1.31	-4.05	3.71



**Figure F-12. Historical FEBA Movement vs Linearly Fitted FEBA Movement
(all destroyed and abandoned tanks, AT wpns, artillery)**

**Table F-12. Historical FEBA Movement vs Linearly Fitted FEBA Movement
(all destroyed and abandoned tanks, AT wpns, artillery)**

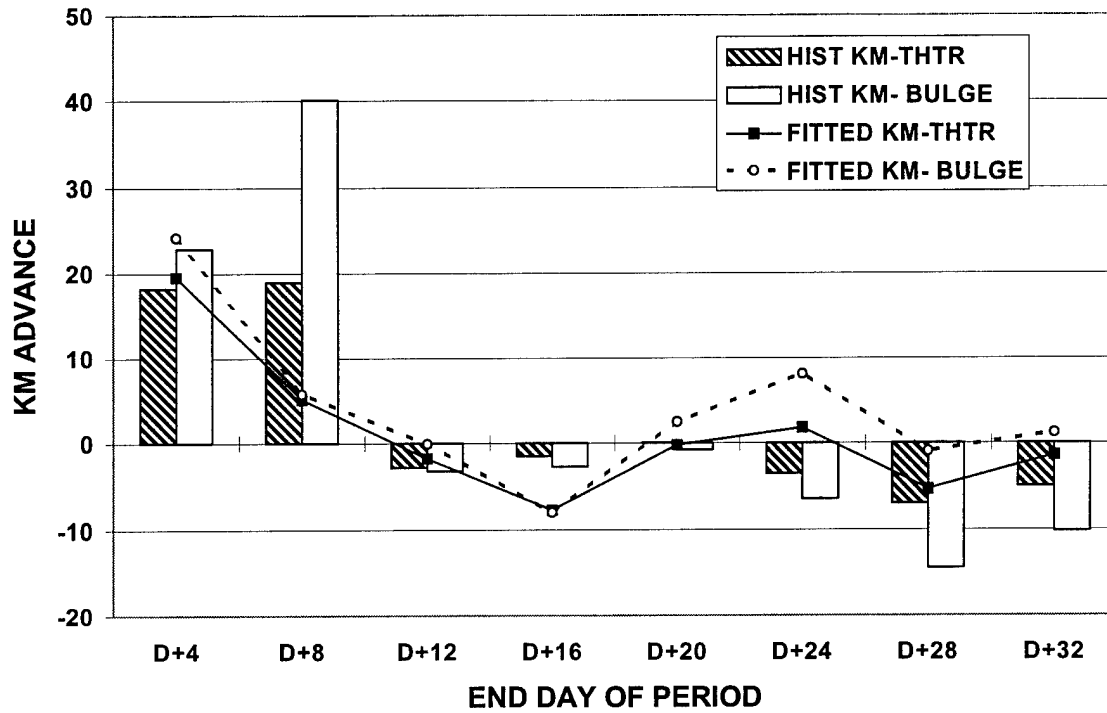
	D+4	D+8	D+12	D+16	D+20	D+24	D+28	D+32
HIST KM-THTR	18.24	18.96	-2.83	-1.5	0.03	-3.56	-7.01	-5.02
FITTED KM-THTR	24.81	9.39	0.58	-2.98	0.09	-3.72	-14.60	-0.54
HIST KM- BULGE	22.83	40.08	-3.23	-2.72	-0.8	-6.45	-14.41	-10.16
FITTED KM- BULGE	27.99	11.52	3.68	-3.08	1.48	-1.17	-11.50	0.58



**Figure F-13. Historical FEBA Movement vs Linearly Fitted FEBA Movement
(all damaged, destroyed, and abandoned tanks, AT wpns, artillery)**

**Table F-13. Historical FEBA Movement vs Linearly Fitted FEBA Movement
(all damaged, destroyed, and abandoned tanks, AT wpns, artillery)**

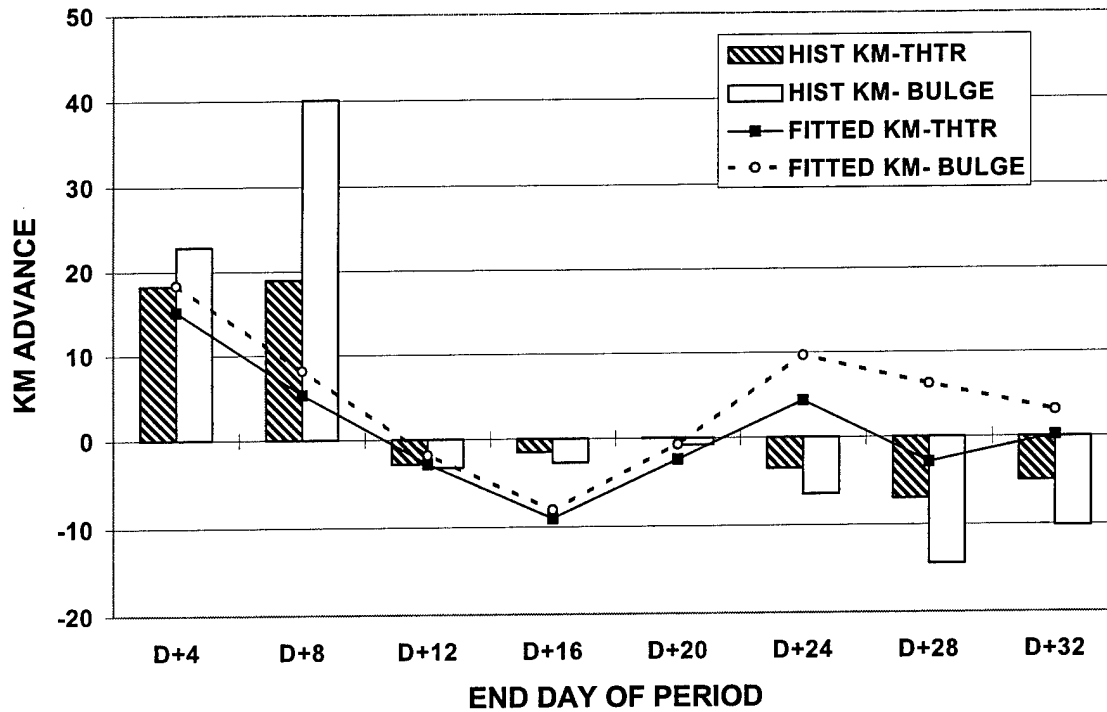
	D+4	D+8	D+12	D+16	D+20	D+24	D+28	D+32
HIST KM-THTR	18.24	18.96	-2.83	-1.5	0.03	-3.56	-7.01	-5.02
FITTED KM-THTR	24.56	5.59	-1.04	-3.99	-2.91	-2.73	-9.21	2.68
HIST KM- BULGE	22.83	40.08	-3.23	-2.72	-0.8	-6.45	-14.41	-10.16
FITTED KM- BULGE	27.37	8.93	0.42	-4.17	-1.90	-1.40	-3.72	3.93



**Figure F-14. Historical FEBA Movement vs Linearly Fitted FEBA Movement
(all destroyed and abandoned tanks)**

**Table F-14. Historical FEBA Movement vs Linearly Fitted FEBA Movement
(all destroyed and abandoned tanks)**

	D+4	D+8	D+12	D+16	D+20	D+24	D+28	D+32
HIST KM-THTR	18.24	18.96	-2.83	-1.5	0.03	-3.56	-7.01	-5.02
FITTED KM-THTR	19.53	5.10	-1.81	-7.73	-0.23	1.79	-5.36	-1.42
HIST KM- BULGE	22.83	40.08	-3.23	-2.72	-0.8	-6.45	-14.41	-10.16
FITTED KM- BULGE	24.18	5.74	-0.11	-8.00	2.46	8.09	-0.96	1.18



**Figure F-15. Historical FEBA Movement vs Linearly Fitted FEBA Movement
(all damaged, destroyed, and abandoned tanks)**

**Table F-15. Historical FEBA Movement vs Linearly Fitted FEBA Movement
(all damaged, destroyed, and abandoned tanks)**

	D+4	D+8	D+12	D+16	D+20	D+24	D+28	D+32
HIST KM-THTR	18.24	18.96	-2.83	-1.5	0.03	-3.56	-7.01	-5.02
FITTED KM-THTR	15.13	5.29	-2.79	-9.05	-2.44	4.30	-2.88	0.15
HIST KM- BULGE	22.83	40.08	-3.23	-2.72	-0.8	-6.45	-14.41	-10.16
FITTED KM- BULGE	18.32	8.14	-1.81	-8.10	-0.74	9.64	6.20	3.09

APPENDIX G

LOGARITHMIC TRENDS BETWEEN FORCE RATIO AND FEBA MOVEMENT

OVERVIEW. This appendix supplements Chapter 5 of the report.

a. Figures G-1 through G-5 show scatter plots with logarithmic trend lines for force ratio versus FEBA advance, as computed from ACSDB data. Tables G-1 through G-5 tabulate the data plotted in the figures. Force ratio is computed from the ACSDB in favor of the German side for each of the five cases associated with the data tabulated in Table 5-5. Each scatter plot case consists of a set of 16 force ratios plotted against 16 associated FEBA advances, for 4-day periods during the campaign. FEBA advance during each 4-day period is expressed in km. Eight of the (FR, FEBA advance) pairs/points in each plot are computed over the full ARCAS theater, and eight are computed over only the ARCAS bulge. Each scatter plot case corresponds to one of the force ratio cases described in Table 2-2. A logarithmic regression trend line is fitted to each of the scatter plots. Each scatter plot also shows the fitted trend line, along with the fitted equation, and the value of R^2 , the coefficient of determination, for the trend line.

b. Figures G-6 through G-10 show the logarithmic-form fitted FEBA movement, plotted at 4-day intervals against the historical (ACSDB) FEBA movement, for each of the five cases with characteristics tabulated in Table 5-5. Tables G-6 through G-10 tabulate the data plotted in the figures. Each fitted FEBA movement is determined by the appropriate fitted equation for that case, using the historical force ratio value for that period and case.

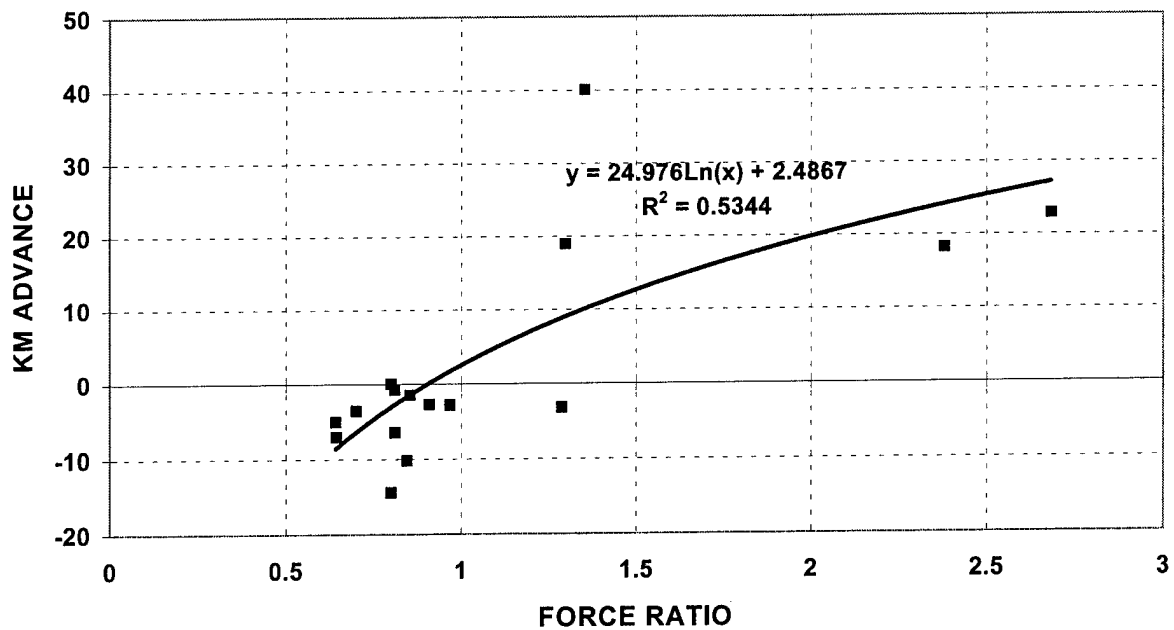
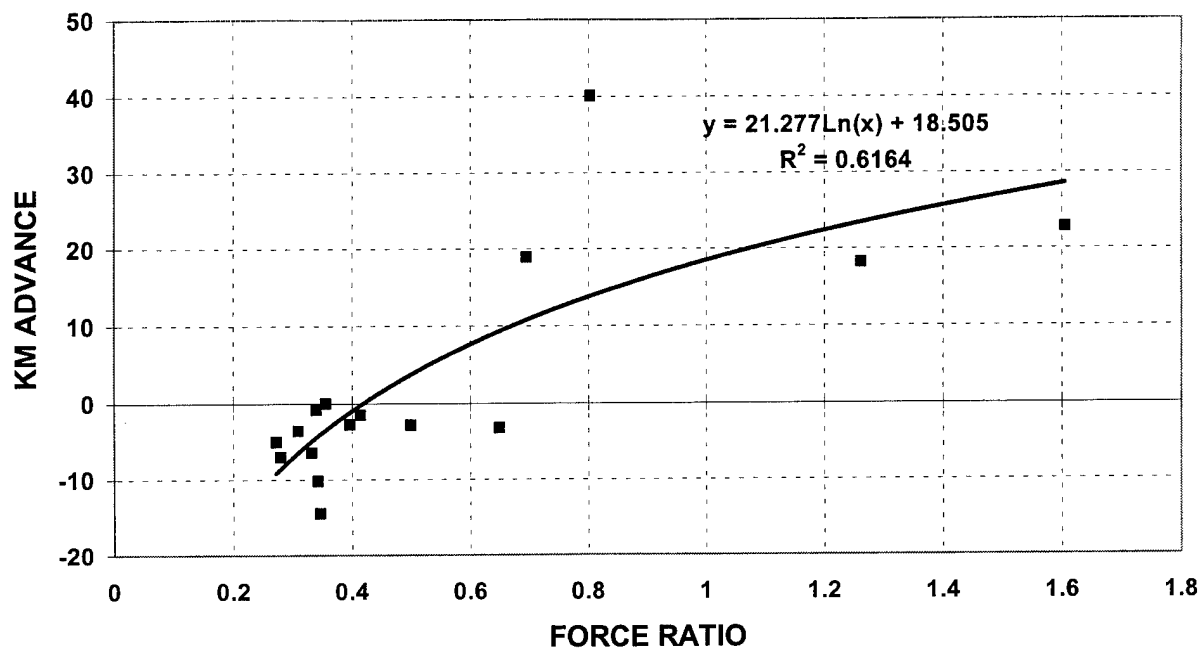


Figure G-1. Logarithmic Fit of Personnel Force Ratio vs FEBA Movement

Table G-1. Logarithmic Fit of Personnel Force Ratio vs FEBA Movement

	D+4	D+8	D+12	D+16	D+20	D+24	D+28	D+32
Theater FR	2.38	1.30	0.97	0.85	0.80	0.70	0.64	0.64
Bulge FR	2.68	1.35	1.29	0.91	0.81	0.81	0.80	0.85
Theater km adv	18.24	18.96	-2.83	-1.5	0.03	-3.56	-7.01	-5.02
Bulge km adv	22.83	40.08	-3.23	-2.72	-0.8	-6.45	-14.41	-10.16



**Figure G-2. Logarithmic Fit of System Force Ratio vs FEBA Movement
(all major systems)**

**Table G-2. Logarithmic Fit of System Force Ratio vs FEBA Movement
(all major systems)**

	D+4	D+8	D+12	D+16	D+20	D+24	D+28	D+32
Theater FR	1.26	0.70	0.50	0.41	0.36	0.31	0.28	0.27
Bulge FR	1.61	0.80	0.65	0.40	0.34	0.33	0.35	0.34
Theater km adv	18.24	18.96	-2.83	-1.5	0.03	-3.56	-7.01	-5.02
Bulge km adv	22.83	40.08	-3.23	-2.72	-0.8	-6.45	-14.41	-10.16

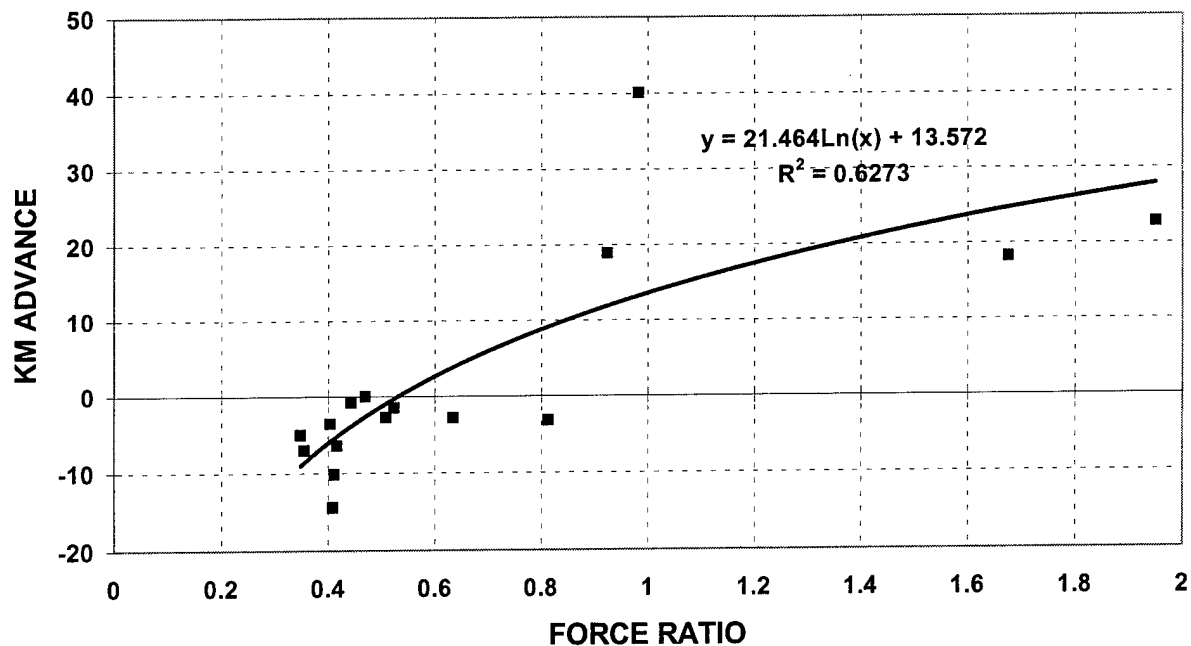
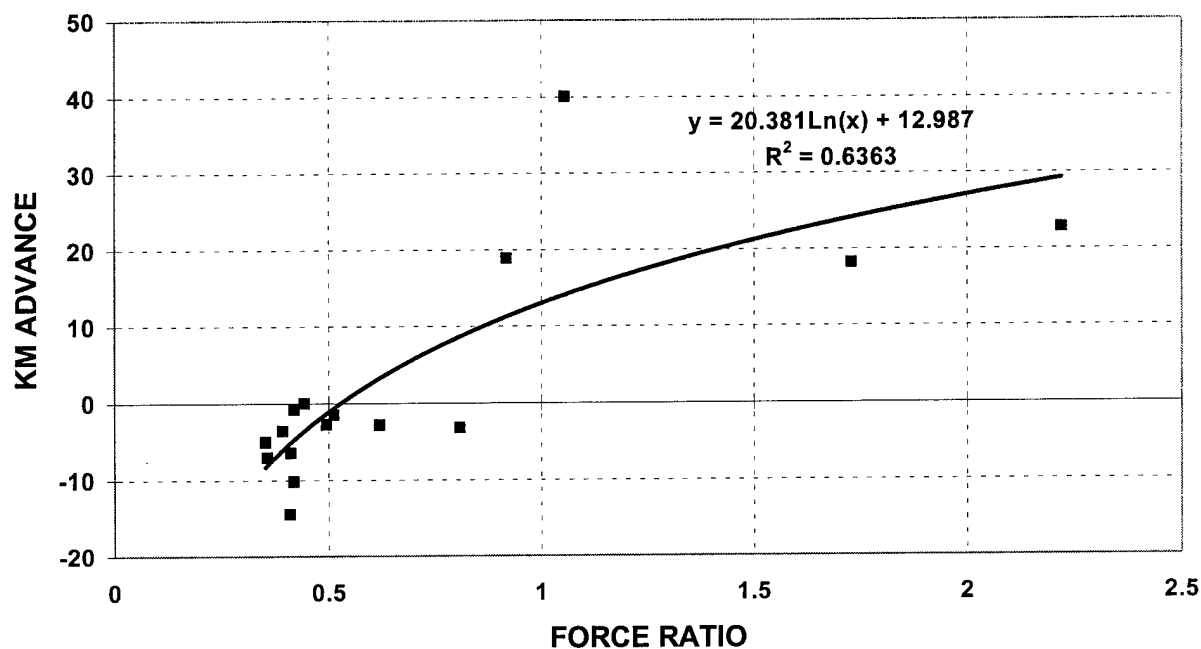


Figure G-3. Logarithmic Fit of System Force Ratio vs FEBA Movement (tanks, AT/Ms, artillery)

Table G-3. Logarithmic Fit of System Force Ratio vs FEBA Movement (tanks, AT/Ms, artillery)

	D+4	D+8	D+12	D+16	D+20	D+24	D+28	D+32
Theater FR	1.68	0.92	0.63	0.52	0.47	0.40	0.36	0.35
Bulge FR	1.95	0.98	0.81	0.51	0.44	0.42	0.41	0.41
Theater km adv	18.24	18.96	-2.83	-1.5	0.03	-3.56	-7.01	-5.02
Bulge km adv	22.83	40.08	-3.23	-2.72	-0.8	-6.45	-14.41	-10.16



**Figure G-4. Logarithmic Fit of System Force Ratio vs FEBA Movement
(tanks, AT weapons, artillery)**

**Table G-4. Logarithmic Fit of System Force Ratio vs FEBA Movement
(tanks, AT wpns, artillery)**

	D+4	D+8	D+12	D+16	D+20	D+24	D+28	D+32
Theater FR	1.73	0.92	0.62	0.51	0.44	0.39	0.36	0.35
Bulge FR	2.22	1.05	0.81	0.50	0.42	0.41	0.41	0.42
Theater km adv	18.24	18.96	-2.83	-1.5	0.03	-3.56	-7.01	-5.02
Bulge km adv	22.83	40.08	-3.23	-2.72	-0.8	-6.45	-14.41	-10.16

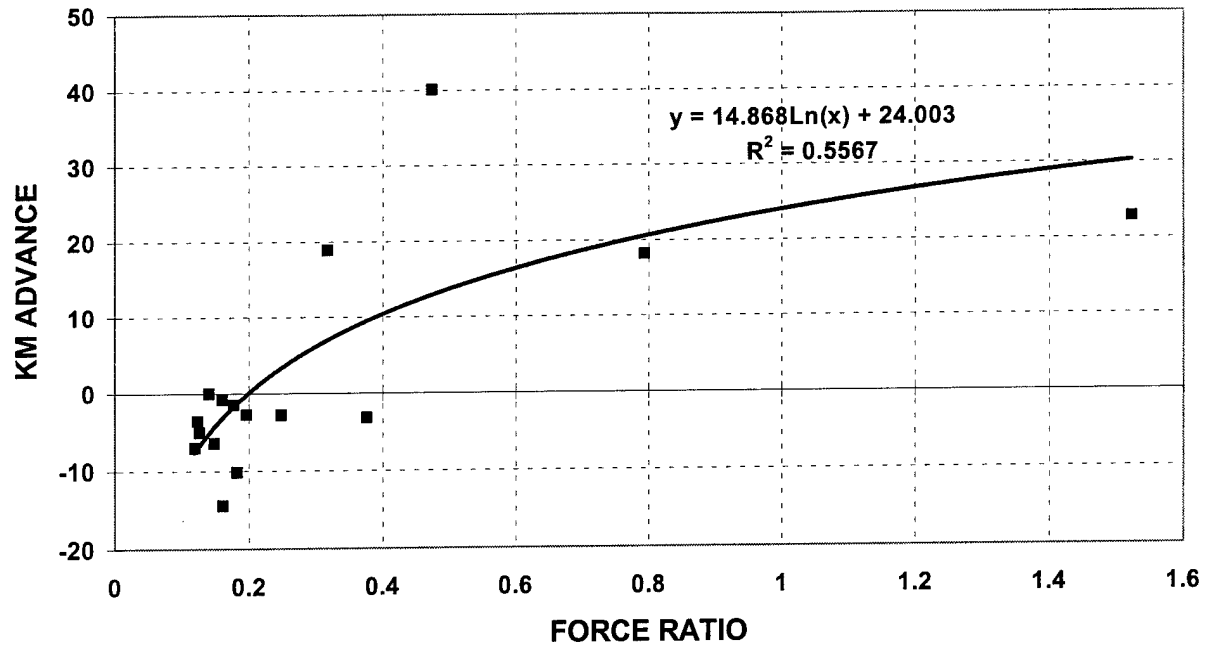


Figure G-5. Logarithmic Fit of System Force Ratio vs FEBA Movement (tanks only)

Table G-5. Logarithmic Fit of System Force Ratio vs FEBA Movement (tanks only)

	D+4	D+8	D+12	D+16	D+20	D+24	D+28	D+32
Theater FR	0.79	0.32	0.25	0.18	0.14	0.12	0.12	0.13
Bulge FR	1.52	0.47	0.38	0.20	0.16	0.15	0.16	0.18
Theater km adv	18.24	18.96	-2.83	-1.5	0.03	-3.56	-7.01	-5.02
Bulge km adv	22.83	40.08	-3.23	-2.72	-0.8	-6.45	-14.41	-10.16

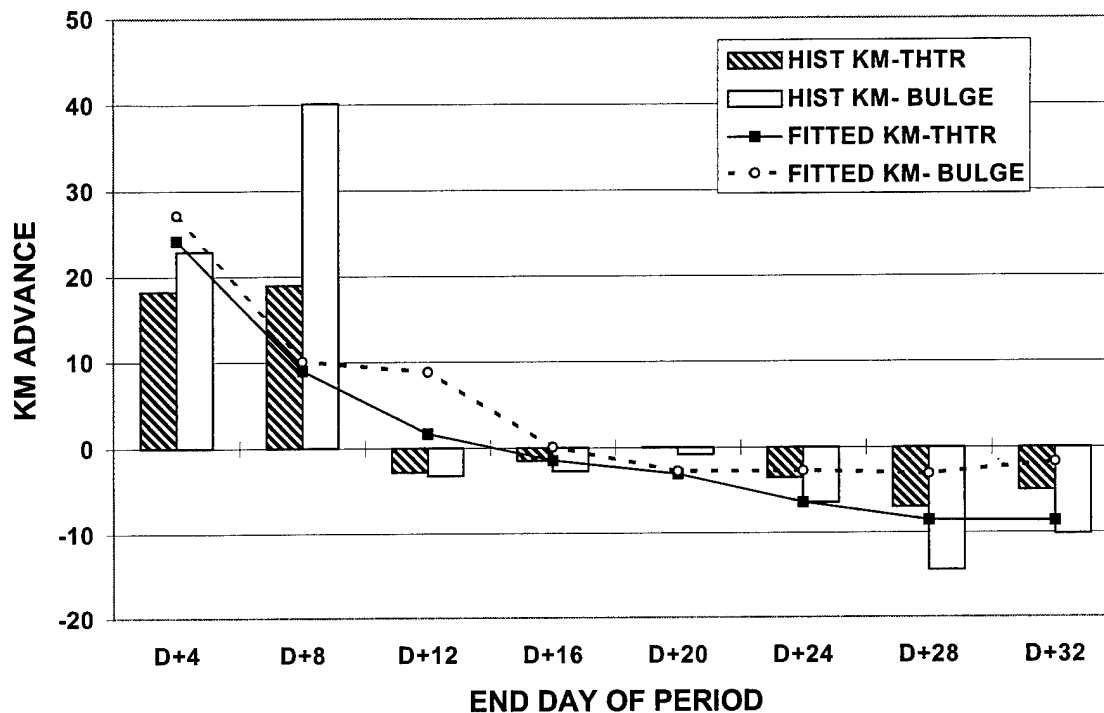


Figure G-6. Historical FEBA Movement vs Logarithmic Fitted FEBA Movement (personnel force ratio)

Table G-6. Historical FEBA Movement vs Logarithmic Fitted FEBA Movement (personnel force ratio)

	D+4	D+8	D+12	D+16	D+20	D+24	D+28	D+32
HIST KM-THTR	18.24	18.96	-2.83	-1.5	0.03	-3.56	-7.01	-5.02
FITTED KM-THTR	24.14	9.00	1.67	-1.45	-3.09	-6.42	-8.54	-8.62
HIST KM- BULGE	22.83	40.08	-3.23	-2.72	-0.8	-6.45	-14.41	-10.16
FITTED KM- BULGE	27.14	10.04	8.81	0.10	-2.78	-2.75	-3.09	-1.72

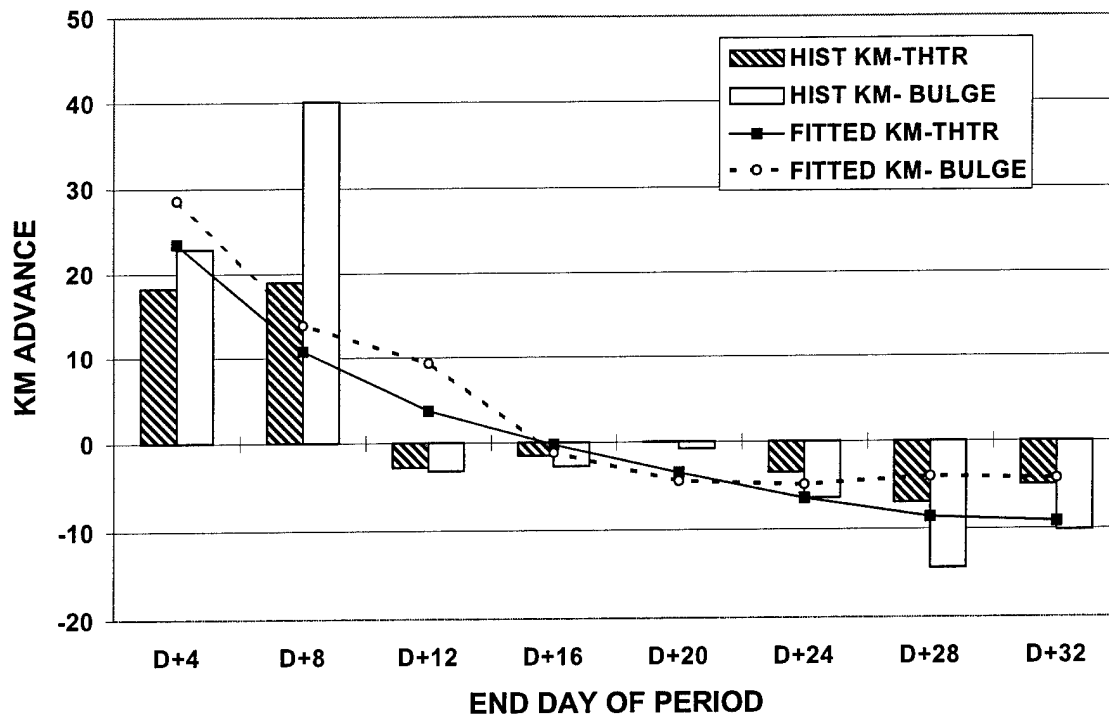


Figure G-7. Historical FEBA Movement vs Logarithmic Fitted FEBA Movement
(all major systems)

Table G-7. Historical FEBA Movement vs Logarithmic Fitted FEBA Movement
(all major systems)

	D+4	D+8	D+12	D+16	D+20	D+24	D+28	D+32
HIST KM-THTR	18.24	18.96	-2.83	-1.5	0.03	-3.56	-7.01	-5.02
FITTED KM-THTR	23.46	10.76	3.71	-0.26	-3.53	-6.48	-8.66	-9.20
HIST KM- BULGE	22.83	40.08	-3.23	-2.72	-0.8	-6.45	-14.41	-10.16
FITTED KM- BULGE	28.57	13.84	9.31	-1.20	-4.51	-4.96	-4.08	-4.32

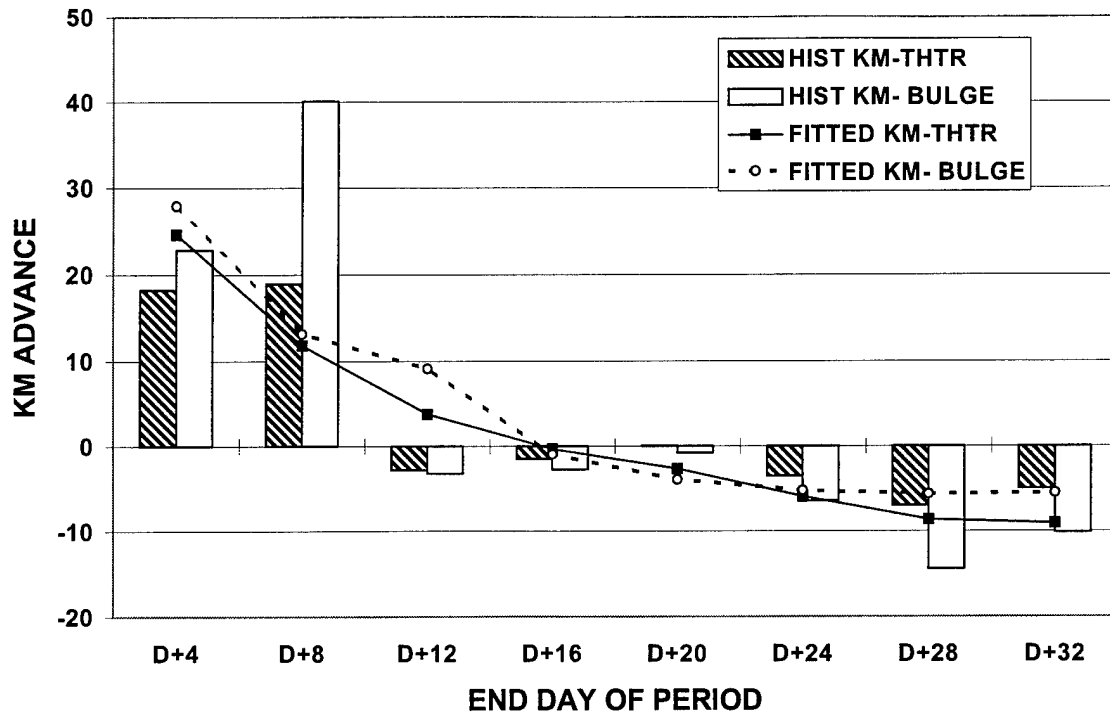


Figure G-8. Historical FEBA Movement vs Logarithmic Fitted FEBA Movement
(all tanks, AT/Ms, artillery)

Table G-8. Historical FEBA Movement vs Logarithmic Fitted FEBA Movement
(all tanks, AT/Ms, artillery)

	D+4	D+8	D+12	D+16	D+20	D+24	D+28	D+32
HIST KM-THTR	18.24	18.96	-2.83	-1.5	0.03	-3.56	-7.01	-5.02
FITTED KM-THTR	24.64	11.85	3.79	-0.34	-2.68	-5.93	-8.66	-9.08
HIST KM- BULGE	22.83	40.08	-3.23	-2.72	-0.8	-6.45	-14.41	-10.16
FITTED KM- BULGE	27.91	13.18	9.13	-0.97	-3.95	-5.25	-5.67	-5.51

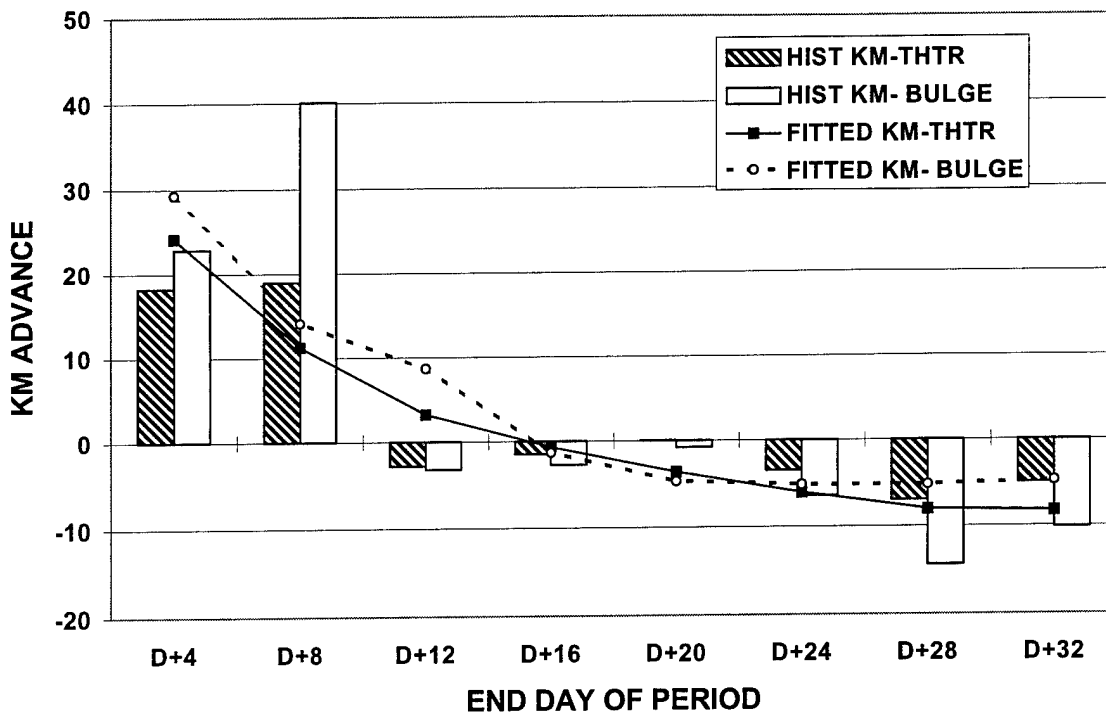


Figure G-9. Historical FEBA Movement vs Logarithmic Fitted FEBA Movement
(all tanks, AT wpns, artillery)

Table G-9. Historical FEBA Movement vs Logarithmic Fitted FEBA Movement
(all tanks, AT wpns, artillery)

	D+4	D+8	D+12	D+16	D+20	D+24	D+28	D+32
HIST KM-THTR	18.24	18.96	-2.83	-1.5	0.03	-3.56	-7.01	-5.02
FITTED KM-THTR	24.15	11.22	3.21	-0.66	-3.65	-6.10	-8.06	-8.29
HIST KM- BULGE	22.83	40.08	-3.23	-2.72	-0.8	-6.45	-14.41	-10.16
FITTED KM- BULGE	29.24	14.06	8.64	-1.34	-4.74	-5.18	-5.23	-4.79

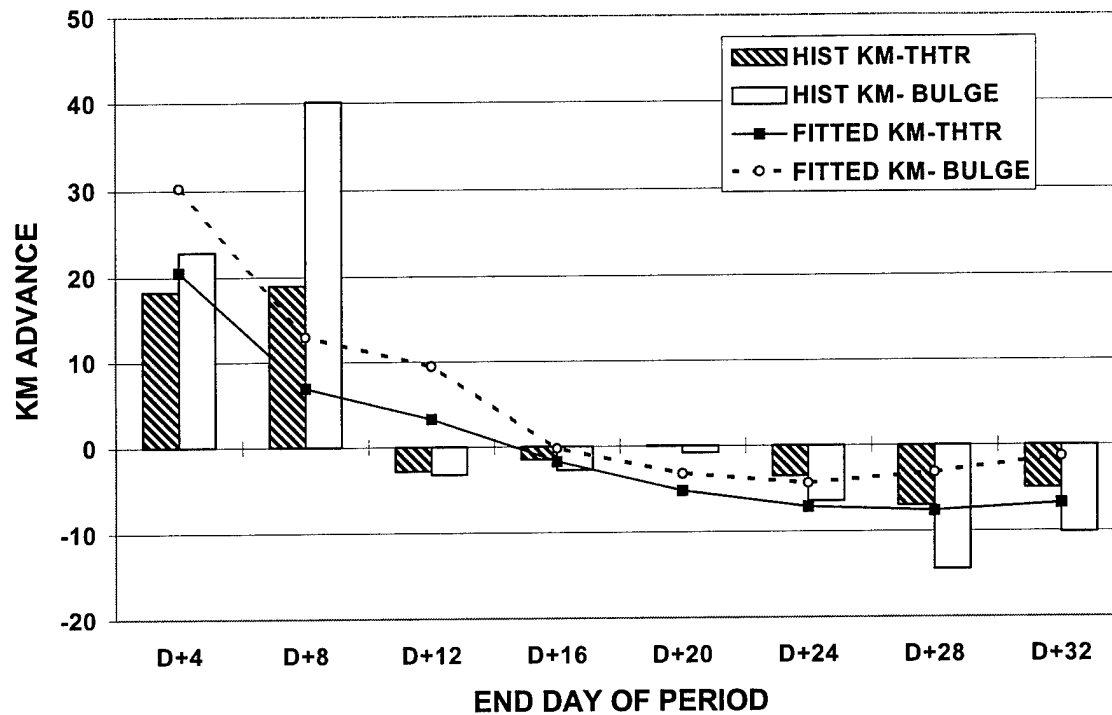


Figure G-10. Historical FEBA Movement vs Logarithmic Fitted FEBA Movement (all tanks)

Table G-10. Historical FEBA Movement vs Logarithmic Fitted FEBA Movement (all tanks)

	D+4	D+8	D+12	D+16	D+20	D+24	D+28	D+32
HIST KM-THTR	18.24	18.96	-2.83	-1.5	0.03	-3.56	-7.01	-5.02
FITTED KM-THTR	20.55	6.92	3.27	-1.74	-5.23	-7.15	-7.65	-6.80
HIST KM- BULGE	22.83	40.08	-3.23	-2.72	-0.8	-6.45	-14.41	-10.16
FITTED KM- BULGE	30.26	12.90	9.46	-0.23	-3.24	-4.40	-3.15	-1.33

GLOSSARY

1. ABBREVIATIONS, ACRONYMS, AND SHORT TERMS

abnd	abandoned
AbnD	airborne division
ACSDB	Ardennes Campaign Simulation Data Base
AC	aircraft
AD	air defense
ADV	advantage factor
APC	armored personnel carrier
arm	armored
ARCAS	Ardennes Campaign Simulation (study)
ARFERR	Ardennes Fractional Exchange Ratio Research
arty	artillery
AT	antitank
ATCAL	Attrition Calibration (process)
ATW	antitank weapon
AT/M	antitank/mortar
avg	average
bde	brigade
CAA	US Army Concepts Analysis Agency
cas	casualties
cbt	combat
CFE	Conventional Forces in Europe (treaty)

CMIA	captured/missing in action
coeff	coefficient
CR	casualty ratio
dmgd	damaged
DNBI	disease and nonbattle injuries
dst	destroyed
eq	equation
expon.	exponential
FBB	Fuehrer Begleit Brigade
FEBA	forward edge of the battle area
FER	fractional exchange ratio
FGB	Fuehrer Grenadier Brigade
FJD	fallschirmjaeger division
FR	force ratio
HERO	Historical Evaluation and Research Organization
hist	historical
ID	infantry division
KCMIA	killed/captured/missing in action
KIA	killed in action
km	kilometer(s)
ln	(natural) logarithm
m	meters
mm	millimeter(s)

MOE	measure(s) of effectiveness
NEA	Northeast Asia
PC	personal computer
pers	personnel
pt	point
PzBde	Panzer brigade
PzGD	Panzer grenadier division
PzD	Panzer division
PzLehrD	(Lehr) Panzer Division
SSPzD	SS Panzer Division
std dev	standard deviation
TACAIR	tactical air
tot	total
TOW	tube launched, optically tracked, wire-guided
thtr	theater
US	United States
UK	United Kingdom
VGD	volks grenadier division
WIA	wounded in action
wpn	weapon
WWII	World War II

2. MODELS, ROUTINES, AND SIMULATIONS

ATCAL	Attrition Calibration - generates simulated combat attrition results, suitable for use in a theater-level simulation
CEM IX	Concepts Evaluation Model IX - a two-sided, fully automated, deterministic model capable of aggregating conventional warfare results as a series of 4-day theater-level cycles
COSAGE	Combat Sample Generator - a two-sided, stochastic, high-resolution (division level) simulation model which simulates a day's combat activity to generate ammunition consumption and equipment and personnel loss data

3. DEFINITIONS

advantage factor

A measure of combat potential of opposing military forces equal to $.5\text{Ln}[(1 - a^2)/(1 - b^2)]$ for the advantage factor favoring side Red, where a = final Blue strength/Blue initial strength, and b = final Red strength/Red initial strength

bulge FER

A bulge FER denotes a FER calculation based on only those committed line units in the ACSBD which comprise the historical "bulge" in the ARCAS scenario theater.

force ratio

For a Red military force opposing a Blue force, the force ratio favoring side Red is defined as: $[\text{total onhand Red items}] / [\text{total onhand Blue items}]$ where the items are personnel or weapon systems in the force. The force ratio favoring side Blue is the reciprocal of this quantity.

fractional exchange ratio

For a Red military force opposing a Blue force, the fractional exchange ratio favoring side Blue in a time period I is defined as

$$(\Delta\text{LOSSES}_{\text{RED}(i)} / \Delta\text{LOSSES}_{\text{BLUE}(i)}) / [\text{Force Ratio}_{(i)}]$$

where $\Delta\text{LOSSES}_{\text{RED}(i)}$ = total losses of specified Red items in period i ,
 $\Delta\text{LOSSES}_{\text{BLUE}(i)}$ = total losses of specified Blue items during period i ,
 and $\text{Force Ratio}_{(i)}$ = force ratio favoring Red at start of period i .

theater FER

A theater FER denotes a FER calculation based on the entire Ardennes theater represented in the ARCAS scenario. A theater FER for a time period is based on assets and losses from all of the ACSDB line units committed to the Ardennes conflict during each 4-day period in the ARCAS scenario.